

REVIEW

## Nonreciprocal hybrid magnetoplasmonics

To cite this article: Dominik Floess and Harald Giessen 2018 *Rep. Prog. Phys.* **81** 116401

View the [article online](#) for updates and enhancements.



**IOP | ebooks™**

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

## Review

# Nonreciprocal hybrid magnetoplasmonics

Dominik Floess and Harald Giessen<sup>1</sup> 

4th Physics Institute and Research Center SCoPE, University of Stuttgart, Stuttgart 70569, Germany

E-mail: [giessen@physik.uni-stuttgart.de](mailto:giessen@physik.uni-stuttgart.de)

Received 20 July 2017, revised 25 June 2018

Accepted for publication 30 July 2018

Published 1 October 2018



CrossMark

Corresponding Editor Professor Masud Mansuripur

**Abstract**

The Faraday effect describes the phenomenon that a magnetized material can alter the polarization state of transmitted light. Interestingly, unlike most light-matter interactions in nature, it breaks Lorentz reciprocity. This exceptional behavior is utilized for applications such as optical isolators, which are core elements in communication and laser systems. While there is high demand for sub-micron nonreciprocal photonic devices, the realization of such systems is extremely challenging as conventional magneto-optic materials only provide weak magneto-optic response within small volumes.

Plasmonics could be a key to overcome this hurdle in the future: over the last years there have been several lines of work demonstrating that different types of metallic nanostructures can be utilized to greatly enhance the magneto-optic response of conventional materials. In this review we give an overview over the state of the art in the field and highlight recent developments on hybrid plasmonic Faraday rotators. Our discussions are mainly focused on the visible and near-infrared wavelength regions and cover both experimental realizations as well as analytical descriptions. Special attention will be paid to recent developments on hybrid plasmonic thin film systems consisting of gold and europium chalcogenides.

Keywords: plasmonics, magneto-optics, Faraday effect, nonreciprocity, nanophotonics

(Some figures may appear in colour only in the online journal)

**Contents**

1. Introduction.....	2	2.4. Comparison of typical magneto-optic materials.....	9
2. Fundamentals.....	3	2.5. Plasmonics.....	11
2.1. Polarization of light.....	3	2.5.1. The dielectric function of metals.....	11
2.2. Light propagation in magneto-optic materials.....	4	2.5.2. Surface plasmon polaritons.....	13
2.2.1. Wave equation.....	4	2.5.3. Localized surface plasmon resonances.....	14
2.2.2. Propagation in anisotropic media.....	4	2.6. Resonances in grating-waveguide structures.....	15
2.2.3. Faraday rotation.....	5	3. State of the field.....	17
2.2.4. Magneto-optic effects in reflection.....	6	3.1. Nonreciprocal photonics.....	18
2.3. Microscopic description of the Faraday effect.....	7	3.2. Magnetoplasmonic antennas.....	19
2.3.1. Classical description.....	7	3.3. Propagating surface-plasmon-polaritons in a magneto-optic environment.....	20
2.3.2. Quantum mechanical description.....	8	3.4. Waveguide-plasmon-polaritons.....	23
		4. Giant Faraday effect in EuSe-Au structures.....	24
		4.1. The waveguide-plasmon-polaritonic approach.....	24

<sup>1</sup> Author to whom any correspondence should be addressed.

4.2. Experimental realization .....	26
4.2.1. Faraday rotation enhancement.....	26
4.2.2. Tunable working wavelength.....	27
4.2.3. Magnetically tunable polarization rotation.....	27
4.3. On the incident angle and film thickness .....	28
4.4. Conclusion .....	29
5. Lorentz force model for magnetoplasmonics .....	29
5.1. Application to EuSe-Au hybrid structures.....	30
5.2. Simplifications and approximations.....	33
5.2.1. Extended model (5 degrees of freedom).....	33
5.2.2. Identifying the gyration of the waveguide.....	34
5.2.3. Simplified model (3 degrees of freedom).....	34
5.3. Conclusion .....	35
6. Giant Faraday effect via induced absorption .....	35
6.1. Realization with EuSe-Au hybrid systems .....	36
6.1.1. Exploiting the regime of induced absorption.....	37
6.1.2. Influence of magnetic field strength.....	38
6.1.3. Simulation of the ellipticity spectra.....	39
6.2. Harmonic oscillator modeling.....	39
6.3. Conclusion .....	41
7. Conclusion and outlook .....	41
Acknowledgments.....	42
References.....	42

## 1. Introduction

The presence of a static magnetic field influences the optical properties of certain materials that are referred to as magneto-optic (MO) materials. This gives rise to several MO effects, such as the Faraday effect [1, 2] in transmission and the MO Kerr effect in reflection [3]. Here, the Faraday effect is of special interest as it is the potentially strongest MO effect. It describes the following phenomenon: when linearly polarized light propagates through a MO material, in the direction of an applied static magnetic field  $B$  the polarization plane of the electromagnetic wave is rotated by the angle

$$\theta = VBz, \quad (1)$$

where  $z$  is the thickness of the material and  $V$  is the material specific Verdet constant. A very intriguing and unique property of the Faraday effect is that time-reversal symmetry and Lorentz reciprocity are broken [4, 5]. As a result, the direction of the polarization rotation is determined by the direction of the applied magnetic field and not by the direction of the wave vector of the incident light. This fact fundamentally distinguishes it from effects such as optical activity [6]. MO effects are actually the only practical way to break Lorentz reciprocity in passive optical systems, since other approaches rely on either nonlinear effects [7] or time modulation [8]. For that reason, Faraday rotators are widely utilized as core elements in nonreciprocal optical devices such as optical isolators [9–11], which require a Faraday rotation of  $45^\circ$ . As such, Faraday rotators are essential components in a multitude

of optical systems, including optical telecommunication networks [12–15] and laser systems [16–18].

Most systems that involve Faraday rotators have recently undergone massive miniaturization. Hence, there is a high demand for down-sized Faraday rotators [19, 20]. Especially in laser systems, optical isolators are often the limiting components when it comes to device miniaturization [19]. However, the miniaturization of optical isolators is extremely challenging, since, according to equation (1), a minimum optical path length must be acquired to reach the required  $45^\circ$  Faraday rotation. A promising approach to overcome this problem is to combine MO materials with plasmonic nanostructures. Over the last years there have been several lines of work demonstrating that plasmonic nanostructures can be utilized to enhance the magneto-optic response of conventional materials significantly.

The first approaches to enhance Faraday rotation were based on MO nanoparticles [21], photonic crystals [22–26], and microcavities [27–29] but were hampered by either weak MO effects or relatively large structure sizes that are also difficult to fabricate. There have also been efforts to enhance the rather weak MO Kerr effect by means of ferromagnetic metallic nanoparticles [30–33], photonic crystals [34] and subwavelength dielectric MO gratings [35]. Considerable attention was received by the work by Chin *et al* in 2013, where the Faraday rotation of a dielectric bismuth-iron-garnet (BIG) thin film is enhanced by attaching a metal grating [36], resulting in Faraday rotation of about  $0.8^\circ$  for a 215 nm thick structure. Here, the strong Faraday rotation originates from the interaction between waveguide resonances in the BIG thin film and localized surface plasmon resonances (LSPRs) in the metal grating [37–39]. At that time, this performance was already a remarkable achievement considering that conventional Faraday rotators require centimeter sized crystals to achieve  $45^\circ$  rotation. However, in the following years there has been a series of works resulting in even thinner Faraday rotators with one order of magnitude larger rotation and more functionality [40–42]. As the performance of these nanoscopic structures is not far away from meeting the requirements of a Faraday isolator, the concept of hybrid magnetoplasmonic crystals could become a key ingredient to realize nonreciprocal photonic devices in a highly integrated environment.

In this review we focus on recent developments on magnetoplasmonic crystal Faraday rotators and put them into the context of nonreciprocal plasmonics and other magneto-optic nanostructures. Our discussion will cover both experimental realizations as well as analytical descriptions. We start with a theoretical primer in section 2, where we introduce the basic physical concepts involved in hybrid magnetoplasmonic systems. This includes magneto-optic effects, plasmonics, and the optical properties of corrugated waveguides. After that, in section 3 we give an overview over the most relevant works in the field of nonreciprocal nanophotonics and magnetoplasmonics. In the sections 4–6 we pay special attention to the most recent experimental and theoretical advances within the realm of hybrid magnetoplasmonics for giant Faraday rotation. Major parts of this review have been adapted with permission from the PhD thesis by Dominik Floess [43] and from the articles [40–42].

## 2. Fundamentals

This section provides a basic discussion of the physical phenomena such as magneto-optic effects, plasmonics, and the modal dispersion of grating-waveguide structures. The concepts introduced here will be basis for the discussions in the following sections. This section has been adapted with permission form [43].

### 2.1. Polarization of light

First, the mathematical foundation and nomenclature for describing the polarization state of classical electromagnetic waves is introduced. The relations compiled in this section are the basis for the discussions throughout this review. The derivations and naming conventions in this section are adapted from Zvezdin and Kotov [44].

A plane electromagnetic wave propagating in  $z$ -direction can be written in the form

$$\mathbf{E}(z, t) = \begin{bmatrix} a \cos(kz - \omega t) \\ b \cos(kz - \omega t + \delta) \end{bmatrix}, \quad (2)$$

where  $\omega$  is the angular frequency and  $k$  the wave number [44]. As an alternative formulation to (2) it is often more convenient to use a notation which is based on the assumption that the actual physical electric field is given by the real part of the complex electric field

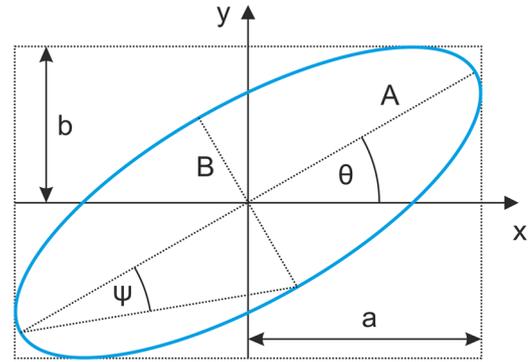
$$\mathbf{E}(z, t) = \tilde{\mathbf{E}} e^{i(kz - \omega t)} \quad (3)$$

with

$$\tilde{\mathbf{E}} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix}. \quad (4)$$

As in the definition of the real electric field (2), the amplitudes  $a$  and  $b$ , as well as the phase angle  $\delta$  are assumed to be real. It is immediately clear that the real part of  $\mathbf{E}(z, t)$  in (3) is equal to  $\mathbf{E}(z, t)$  in (2).

The electromagnetic wave described by the general expressions (3) and (4) can be interpreted as the superposition of two waves, where one is oscillating only in  $x$  direction and the other in  $y$  direction. However, depending on the phase  $\delta$  between the two partial waves, the resulting field vector (3) performs an elliptical motion. This is illustrated by figure 1. It shows a parametric plot of the real part of the electric field, where the varied parameter is the propagator term  $\omega t - kz$ . The resulting shape is an ellipse, which is fully characterized by the two angles  $\theta$  and  $\psi$ . While  $\theta$  determines the *tilt* of the polarization ellipse with respect to the  $x$ -axis, the angle  $\psi$  describes the degree of *ellipticity*. If  $\psi = 0$ , the wave is linearly polarized, whereas for  $|\psi| = \pi$  the wave is circularly polarized. Furthermore, the handedness of the circular polarization can be expressed via the sign of  $\psi$ , i.e.  $\psi = +(-)\pi$  corresponds to right(left)-handed circular polarization. The two angles are related to the coefficients in (2) in the following way [44]:



**Figure 1.** Polarization ellipse of a plane electromagnetic wave with a tilting angle  $\theta$  and ellipticity angle  $\psi$ .

$$\tan(2\theta) = \frac{2 \operatorname{Re}(\chi)}{1 - |\chi|^2} \quad (5a)$$

$$\tan(2\psi) = \frac{2 \operatorname{Im}(\chi)}{1 - |\chi|^2} \quad (5b)$$

with

$$\chi \equiv \frac{E_y}{E_x} = \frac{b e^{i\delta}}{a}. \quad (6)$$

For the important case of  $|\chi| \ll 1$  the equations (5) become

$$\theta \approx \operatorname{Re}(\chi) \quad (7a)$$

$$\psi \approx \operatorname{Im}(\chi). \quad (7b)$$

While  $\theta$  and  $\psi$  fully determine the polarization state of a plane wave they are not directly accessible in experiment. The reason is that the electric and magnetic fields of a light wave usually cannot be measured directly and have to be derived from time-averaged intensity measurements. For that reason, the *Stokes formalism* is an important alternative description of polarization states as it is only based on time-averaged intensities. In this formalism the polarization state of light can be expressed by means of the four Stokes parameters [44, 45]

$$S_0 = I_x + I_y \quad (8a)$$

$$S_1 = I_x - I_y \quad (8b)$$

$$S_2 = I_{+45^\circ} + I_{-45^\circ} \quad (8c)$$

$$S_3 = I_R + I_L. \quad (8d)$$

The quantities  $I_x$  and  $I_y$  correspond to the time averaged intensity of the  $x$ - and  $y$ -polarized components of the light. Furthermore,  $I_{+45^\circ}$  and  $I_{-45^\circ}$  denote the time averaged intensity measured after the light passes a perfect polarizer tilted by  $+45^\circ$  and  $-45^\circ$ , respectively. Similarly,  $I_R$  and  $I_L$  are the time-averaged intensities of the right-handed and left-handed components of the plane wave. The time-averaged description of the Stokes formalism also allows to take the degree of polarization

$$\Pi = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad (0 \leq \Pi \leq 1) \quad (9)$$

into account. Since every plane wave of the form (2) has a well defined polarization state, the degree of polarization is  $\Pi = 1$ . On the other hand, in the case of unpolarized light, the measurement of the Stokes parameters would lead to  $\Pi = 0$ . This is the typical situation for incoherent light emitted by thermal light sources with rapidly and randomly changing field amplitude vectors. If the degree of polarization is in between the two extremes, the light is partially polarized.

In the case of  $\Pi = 1$ , the Stokes parameters can be related to the quantities in equation (2) in the following way:

$$S_0 = a^2 + b^2 = \sqrt{S_1^2 + S_2^2 + S_3^2} \quad (10a)$$

$$S_1 = a^2 - b^2 = S_0 \cos(2\psi) \cos(2\theta) \quad (10b)$$

$$S_2 = 2ab \cos \delta = S_0 \cos(2\psi) \sin(2\theta) \quad (10c)$$

$$S_3 = 2ab \sin \delta = S_0 \sin(2\psi). \quad (10d)$$

Dividing equation (10c) by (10b) and equation (10d) by (10a) yields the angles

$$\theta = \frac{1}{2} \arctan \left( \frac{S_2}{S_1} \right) \quad (11a)$$

$$\psi = \frac{1}{2} \arcsin \left( \frac{S_3}{S_0} \right). \quad (11b)$$

In the case of a complex electric field, the Stokes parameters can be derived using

$$S_0 = E_x E_x^* + E_y E_y^* \quad (12a)$$

$$S_1 = E_x E_x^* - E_y E_y^* \quad (12b)$$

$$S_2 = E_x E_y^* + E_x^* E_y \quad (12c)$$

$$S_3 = i(E_x E_y^* - E_x^* E_y). \quad (12d)$$

## 2.2. Light propagation in magneto-optic materials

The characteristic property of magneto-optic (MO) materials is that their associated permittivity tensor possesses magnetic field induced off-diagonal elements which are antisymmetric. In this section it will be elaborated on how this characteristic structure of the permittivity tensor influences the propagation of light inside magneto-optic materials. The most important outcome of this section is that the *Faraday effect* is directly proportional to the off-diagonal elements in the permittivity tensor.

A key element of the following discussions is the Helmholtz wave equation, which describes the evolution of an electromagnetic wave inside a medium with a given permittivity tensor. At first, this relation is derived from the macroscopic Maxwell equations in matter and then applied to

both anisotropic and isotropic MO materials. The derivations in this section are based on the works by Jackson [46], Yariv [47], as well as by Zvezdin and Kotov [44].

**2.2.1. Wave equation.** Electromagnetic fields in a medium can be described by the macroscopic Maxwell equations

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad (13a)$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \quad (13b)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (13c)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (13d)$$

together with the constitutive equations

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad (14a)$$

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}. \quad (14b)$$

In the following, it is assumed that no sources are present, i.e.  $\mathbf{J} = 0$  and  $\rho = 0$ . Furthermore, the time-harmonic ansatz

$$\Psi(\mathbf{r}, t) = \Psi(\mathbf{r}) e^{-i\omega t} \quad (\Psi = \mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}) \quad (15)$$

turns the equations (13) and (14) into

$$\nabla \times \mathbf{E}(\mathbf{r}) - i\omega \mathbf{B}(\mathbf{r}) = 0 \quad (16a)$$

$$\nabla \times \mathbf{H}(\mathbf{r}) + i\omega \mathbf{D}(\mathbf{r}) = 0 \quad (16b)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0 \quad (16c)$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}) = 0 \quad (16d)$$

and

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0 \varepsilon(\omega) \mathbf{E}(\mathbf{r}) \quad (17a)$$

$$\mathbf{B}(\mathbf{r}) = \mu_0 \mu(\omega) \mathbf{H}(\mathbf{r}). \quad (17b)$$

In general, both the relative permittivity  $\varepsilon(\omega)$  and the relative permeability  $\mu(\omega)$  are tensorial quantities and frequency dependent. However, at optical frequencies we can assume that  $\mu = 1$  [1]. By applying the operator  $\nabla \times$  to equation (16a) and by using the relations  $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \Delta \mathbf{E}$  and (16d), the Helmholtz wave equation

$$\Delta \mathbf{E}(\mathbf{r}) + \frac{\omega^2}{c_0^2} \varepsilon(\omega) \mathbf{E}(\mathbf{r}) = 0 \quad (18)$$

is obtained. This differential equation describes the evolution of an electromagnetic wave inside a medium with a given dielectric tensor  $\varepsilon(\omega)$ .

**2.2.2. Propagation in anisotropic media.** The effect of a static magnetic field on a MO material can be expressed by means of the dielectric tensor of the material. In the following we consider an anisotropic MO material with an applied static magnetic field  $\mathbf{B}$  pointing in  $z$  direction. Furthermore,

we assume that the principal dielectric axes of the material are  $\mathbf{e}_x$ ,  $\mathbf{e}_y$ , and  $\mathbf{e}_z$ . In this case, the dielectric tensor takes the form

$$\varepsilon = \begin{pmatrix} \varepsilon_x & +ig(B) & 0 \\ -ig(B) & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}. \quad (19)$$

The magnetic-field induced off-diagonal elements of  $\varepsilon$  are the characterizing property of MO materials and the origin for all MO effects, such as the Faraday effect or the MO Kerr effect. The quantity  $g$  is often referred to as *gyration coefficient*. For small magnetic fields the gyration coefficient is usually proportional to  $B$ , whereas for larger magnetic fields saturation sets in. This is also the case for the materials EuSe and EuS, which are utilized in the MO nanostructures presented in later sections. It should be noted that in the absence of absorption the permittivity tensor must be hermitian, i.e.  $\varepsilon_{ij} = \varepsilon_{ji}^*$ , which implies that  $g$  must be real in this case [1].

To solve the wave equation (18) we make the ansatz for a plane wave propagating in  $z$ -direction:

$$\mathbf{E}(\mathbf{r}) = \tilde{\mathbf{E}}e^{ikz}. \quad (20)$$

Since we can assume that the amplitude vector  $\tilde{\mathbf{E}}$  lies within the  $xy$  plane, we omit its  $z$  component in the following derivations. Hence, we can reduce the permittivity tensor to

$$\varepsilon = \begin{pmatrix} \varepsilon_x & +ig \\ -ig & \varepsilon_y \end{pmatrix}. \quad (21)$$

Inserting (20) and (21) into the wave equation (18) leads to the eigenvalue problem

$$\varepsilon\tilde{\mathbf{E}} = \lambda\tilde{\mathbf{E}} \quad (22)$$

with

$$\lambda = k^2 \frac{c_0^2}{\omega^2}. \quad (23)$$

Solving the eigenvalue problem yields the eigenvalues

$$\lambda_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \mp \sqrt{g^2 + \frac{(\varepsilon_x - \varepsilon_y)^2}{4}} \quad (24)$$

and the eigenvectors

$$\tilde{\mathbf{E}}^{(1,2)} = \begin{bmatrix} \left( \frac{\varepsilon_x - \varepsilon_y}{2} \mp \sqrt{g^2 + \frac{(\varepsilon_x - \varepsilon_y)^2}{4}} \right) \frac{1}{-ig} \\ 1 \end{bmatrix}. \quad (25)$$

Consequently, the general solution of (18) is then given by the superposition

$$\mathbf{E}(\mathbf{r}) = A\tilde{\mathbf{E}}^{(1)}e^{ik_1z} + B\tilde{\mathbf{E}}^{(2)}e^{ik_2z} \quad (26)$$

with

$$k_{1,2} = \frac{\omega}{c_0} \sqrt{\lambda_{1,2}}. \quad (27)$$

The coefficients  $A$  and  $B$  are determined by the initial condition. For example, the case of  $x$ -polarized incident light implies that  $E_y(z=0) = 0$  and  $B = -A$ , which leads to

$$\mathbf{E}(\mathbf{r}) = A \left[ \tilde{\mathbf{E}}^{(1)}e^{ik_1z} - \tilde{\mathbf{E}}^{(2)}e^{ik_2z} \right]. \quad (28)$$

Similarly,  $y$ -polarized incident light is described by  $E_x(z=0) = 0$  and implies  $B = -A\tilde{E}_x^{(1)}/\tilde{E}_x^{(2)}$ . This leads to a wave propagation of the form

$$\mathbf{E}(\mathbf{r}) = A \left[ \tilde{\mathbf{E}}^{(1)}e^{ik_1z} - \frac{\tilde{E}_x^{(1)}}{\tilde{E}_x^{(2)}}\tilde{\mathbf{E}}^{(2)}e^{ik_2z} \right]. \quad (29)$$

In both cases, the coefficient  $A$  is an arbitrary normalization parameter depending on the incident intensity. The equations (28) and (29) are utilized in section 5.1 where the magneto-optical response of hybrid plasmonic nanostructures is described by means of a birefringent effective medium.

**2.2.3. Faraday rotation.** Many bulk magneto-optic materials, such as EuSe and EuS are not birefringent, i.e. the diagonal elements of the dielectric function are equal. In this case, the relative permittivity tensor (21) simplifies to

$$\varepsilon(\omega) = \begin{pmatrix} \varepsilon_x & +ig \\ -ig & \varepsilon_x \end{pmatrix} \quad (30)$$

and the solution of the eigenvalue problem (22) turns into

$$\lambda_{1,2} = \varepsilon_x \pm g \quad (31)$$

and

$$\tilde{\mathbf{E}}^{(1,2)} = \begin{pmatrix} \pm i \\ 1 \end{pmatrix}. \quad (32)$$

Furthermore, the absolute value of the gyration is typically significantly smaller than the diagonal permittivity, i.e.  $|g|/|\varepsilon_x| \ll 1$ . In this approximation, we can write the two propagation constants  $k_1$  and  $k_2$  as

$$k_{1,2} = \frac{\omega}{c_0} \sqrt{\varepsilon_x \pm g} \approx \kappa \mp \gamma, \quad (33)$$

with

$$\kappa = \frac{\omega}{c_0} \sqrt{\varepsilon_x} \quad (34)$$

$$\gamma = -\frac{1}{2} \frac{\omega}{c_0} \frac{g}{\sqrt{\varepsilon_x}}. \quad (35)$$

From this, through equation (28) for  $x$ -polarized incident light, we obtain

$$\mathbf{E}(\mathbf{z}, t) = A e^{i(\kappa z - \omega t)} \left[ \begin{pmatrix} +i \\ 1 \end{pmatrix} e^{-i\gamma z} - \begin{pmatrix} -i \\ 1 \end{pmatrix} e^{+i\gamma z} \right]. \quad (36)$$

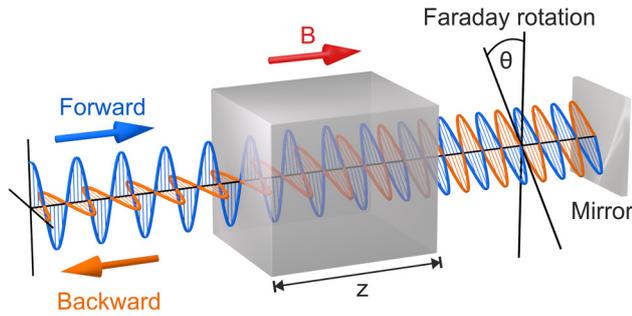
To evaluate how the polarization state of the wave (36) evolves, we can apply the relations (12) to derive the corresponding Stokes parameters

$$S_0 = +4|A|^2 \cosh(2 \operatorname{Im}\{\gamma\}z) \quad (37a)$$

$$S_1 = +4|A|^2 \cos(2 \operatorname{Re}\{\gamma\}z) \quad (37b)$$

$$S_2 = -4|A|^2 \sin(2 \operatorname{Re}\{\gamma\}z) \quad (37c)$$

$$S_3 = -4|A|^2 \sinh(2 \operatorname{Im}\{\gamma\}z). \quad (37d)$$



**Figure 2.** Illustration of the Faraday effect for the case of a real gyration  $g$  and negligible absorption. When light propagates through a magneto-optical material along the direction of the applied magnetic field (forward direction) the polarization plane gets rotated by the angle  $\theta$ . Backward propagating light is rotated by the same angle and in the same direction (relative to the magnetic field).

Furthermore, in the limit of a relatively weak gyration, i.e. for  $|\gamma|z \ll 1$ , the relations (11) and (35) yield

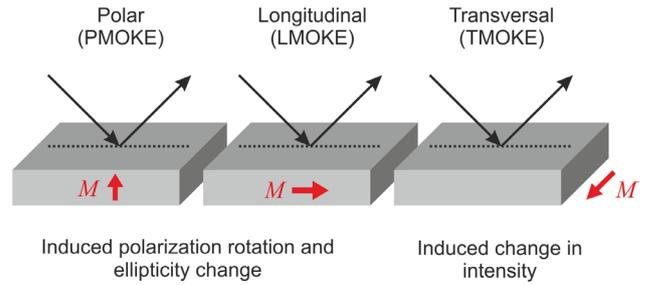
$$\theta = \frac{1}{2} \frac{\omega}{c_0} \operatorname{Re} \left\{ \frac{g}{\sqrt{\varepsilon_x}} \right\} z \quad \text{and} \quad (38a)$$

$$\psi = \frac{1}{2} \frac{\omega}{c_0} \operatorname{Im} \left\{ \frac{g}{\sqrt{\varepsilon_x}} \right\} z. \quad (38b)$$

From this we see that in the case of low material losses and a small imaginary part of  $g$ , the ellipticity  $\psi$  stays small, while the tilting angle  $\theta$  increases linearly with the propagation distance  $z$ . Since the gyration  $g$  is proportional to the applied magnetic field  $B$ , it is very common to write equation (38a) in the alternative form

$$\theta = VBz, \quad (39)$$

where the proportionality factor  $V$  is commonly referred to as *Verdet constant*. However, the term *constant* is a bit misleading, since  $V$  is actually frequency dependent. This is immediately clear by considering that equation (38a) contains both the factor  $\omega$  and the gyration  $g$ , which is also frequency dependent. The magnetic field induced polarization rotation described by the expression (39) is called *Faraday effect* and it is illustrated by the blue wave in figure 2. It is important to realize that the magnetic field induced polarization rotation breaks the *time-reversal symmetry*. The result of this symmetry breaking becomes clear when a mirror is added behind the MO material as indicated in figure 2. In relation to the magnetic field vector, the backward propagating orange light wave is rotated in the same direction and by the same angle as the forward propagating blue wave. Hence, the polarization states of the blue and orange waves on the left-hand side are different. Such a nonreciprocal behavior is not found in other linear and static systems [48]. For example, although an optically active medium [6] can also induce a polarization rotation of transmitted light, any back-reflected wave would rotate back to the polarization state of the incoming wave. In other words, in the case of an optically active medium the direction of polarization rotation is determined by the direction of the wave vector, whereas the direction of Faraday rotation is determined by the direction of the magnetic field.



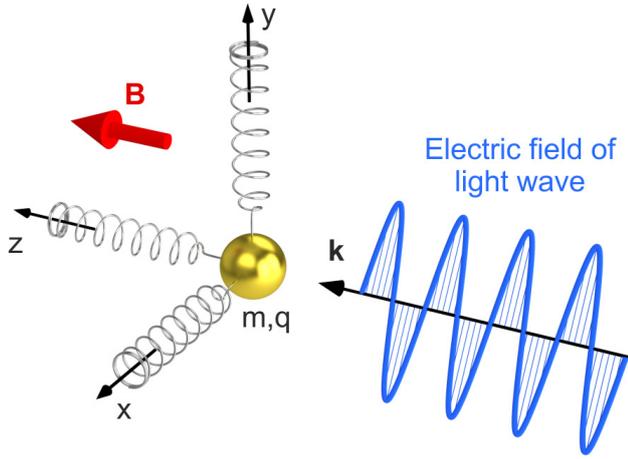
**Figure 3.** Illustration of the different manifestations of the magneto-optic Kerr effect (MOKE). While polar and longitudinal MOKE describe a magnetically induced polarization rotation and ellipticity change, in the case of transversal MOKE a magnetization dependent modulation of the reflected intensity occurs.

This nonreciprocal nature [4, 49] of the Faraday effect directly leads to its most prominent application, namely optical isolation: if the rotation angle  $\theta$  is equal to  $45^\circ$ , the polarizations of the forward and backward propagating waves on the left-hand side of figure 2 are perpendicular. In this case, the addition of one vertically aligned polarizer on the left-hand side of the MO material and another polarizer, which is tilted by  $45^\circ$ , on the right-hand side of the MO material, allows light to pass the system only in forward direction. Light traveling through the system in backward direction is blocked by the polarizer on the left side.

**2.2.4. Magneto-optic effects in reflection.** The Faraday effect is only one of several phenomena arising from the magnetically induced off-diagonal elements in  $\epsilon(\omega)$ . For example, also the reflection behavior of a MO material can change depending on the applied magnetic field. This phenomenon is referred to as magneto-optic Kerr effect (MOKE). Since the MOKE does rely on reflection rather than propagation it cannot only be observed for transparent materials but also for opaque materials such as ferromagnetic metals.

Although the work presented in this review concentrates on the Faraday effect, in the following a brief overview of the different types of the MOKE is given. As illustrated in figure 3, the different types are classified depending on the direction of the magnetization in relation to the plane of incidence and the material surface [44]. The black arrows indicate the wavevectors of the incident and reflected light. If the magnetization is perpendicular to the material surface, there occurs a magnetically induced polarization rotation as well as a change in ellipticity of the reflected light. This effect is referred to as the polar MOKE. Similarly, if the magnetization is parallel to the surface the longitudinal MOKE occurs. Here, the magnetization also induces a polarization rotation and an ellipticity change. If the magnetization is perpendicular to the plane of incidence, the intensity of the reflected light changes with the magnitude of magnetization. This phenomenon is called transverse MOKE.

Since the polarization rotation of the MOKE only scales with the off-diagonal elements of the permittivity tensor, the MOKE is generally much weaker than the Faraday effect, which also scales with the optical path length through the material. For example, although Fe, Co and Ni possess an



**Figure 4.** Schematic drawing of the mechanical oscillator model for magneto-optic solids. It corresponds to the standard Lorentz oscillator model for dielectrics, but with the addition of a static magnetic field, which exerts a Lorentz force on the bound electrons.

extremely large Verdet constant (see section 2.4) the rotation angles arising from the polar MOKE are usually less than 1 deg in the visible [44].

### 2.3. Microscopic description of the Faraday effect

In the section 2.2 it has been shown that the magneto-optic response of a material can be described phenomenologically by the anti-symmetric off-diagonal elements of the material's permittivity tensor. However, the microscopic mechanisms which give rise to these off-diagonal elements were not discussed. In this section the microscopic origin of magneto-optic effects is explained by means of both a classical and a quantum mechanical approach.

**2.3.1. Classical description.** The microscopic origin of the magnetically induced off-diagonal elements in (19) can be understood intuitively in the picture of an extended Lorentz oscillator model. This model and the calculations in this section are largely based on the descriptions given in [50–52]. However, the model is presented in a slightly generalized way, which allows a more direct link to the development of the oscillator model of magnetoplasmonics presented in section 5. As in the standard version of the Lorentz oscillator model for dielectrics [52], the bound electrons of a solid are treated as classical mechanical oscillators, which are suspended by three springs. This is illustrated by figure 4. Furthermore, as each oscillator is assumed to have charge  $q$ , it is driven by the harmonically oscillating electric field  $\mathbf{E}(t) = \mathbf{E}e^{-i\omega t}$  of the light wave. The mass and the displacement vector of the oscillator are denoted by  $m$  and  $\mathbf{r} = (x, y, z)^T$ , respectively. To account for magneto-optic effects, in the extended version of the model a static magnetic field  $\mathbf{B}$  is assumed to be present, which additionally exerts the *Lorentz force*

$$\mathbf{F}_L = q\dot{\mathbf{r}} \times \mathbf{B} \quad (40)$$

on the oscillators. For the following discussions we assume that the magnetic field points in  $z$  direction. In this case, the

Lorentz force is given by  $\mathbf{F}_L = qB(\dot{y}, -\dot{x}, 0)^T$  and the resulting equations of motion are

$$m\ddot{x} = -d_x x - 2\gamma_x \dot{x} + qB\dot{y} + qE_x \exp(-i\omega t) \quad (41a)$$

$$m\ddot{y} = -d_y y - 2\gamma_y \dot{y} - qB\dot{x} + qE_y \exp(-i\omega t) \quad (41b)$$

$$m\ddot{z} = -d_z z - 2\gamma_z \dot{z} + qE_z \exp(-i\omega t). \quad (41c)$$

The quantities  $\{d_x, d_y, d_z\}$  and  $\{\gamma_x, \gamma_y, \gamma_z\}$  denote the stiffness and damping coefficients of the springs. To solve the equations, we start by making a time harmonic ansatz for the oscillator displacement, i.e.

$$\mathbf{r}(t) = \mathbf{r}_0 \exp(-i\omega t), \quad (42)$$

which turns the equations (41) into

$$(-\omega^2 + \Omega_x^2 - 2i\Gamma_x \omega)x_0 + i\omega\omega_c y_0 = \frac{q}{m}E_x \quad (43a)$$

$$(-\omega^2 + \Omega_y^2 - 2i\Gamma_y \omega)y_0 - i\omega\omega_c x_0 = \frac{q}{m}E_y \quad (43b)$$

$$(-\omega^2 + \Omega_z^2 - 2i\Gamma_z \omega)z_0 = \frac{q}{m}E_z, \quad (43c)$$

with  $\Omega_i^2 = d_i/m$  and  $\Gamma_i = \gamma_i/m$  ( $i = x, y, z$ ) and  $\omega_c = qB/m$ . The equations (43) can be reformulated in the matrix form, which yields

$$M(\omega) \mathbf{r}_0 = \frac{q}{m} \mathbf{E}, \quad (44)$$

with

$$M(\omega) = \begin{pmatrix} M_x & +i\omega\omega_c & 0 \\ -i\omega\omega_c & M_y & 0 \\ 0 & 0 & M_z \end{pmatrix} \quad (45)$$

and  $M_i(\omega) = -\omega^2 + \Omega_i^2 - 2i\Gamma_i \omega$ , ( $i = x, y, z$ ). Assuming that  $n$  is the oscillator density in the medium, we can express the macroscopic electronic polarization as

$$\mathbf{P} = nq\mathbf{r}_0 = \frac{nq^2}{m} M^{-1} \mathbf{E}. \quad (46)$$

Furthermore, the comparison with

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \quad (47)$$

allows to identify the electronic susceptibility

$$\chi(\omega) = \omega_q^2 M(\omega)^{-1}, \quad (48)$$

with

$$\omega_q = \sqrt{\frac{nq^2}{\varepsilon_0 m}}. \quad (49)$$

The optical response of a material is usually expressed by the electric permittivity tensor  $\varepsilon = \mathbb{1} + \chi$ , where  $\mathbb{1}$  denotes the identity matrix  $\text{diag}(1, 1, 1)$ . To also account for a (scalar) background susceptibility due to other off-resonant electronic polarizations [52], we write the permittivity in the more general form  $\varepsilon = \varepsilon_\infty \mathbb{1} + \chi$ , where  $\varepsilon_\infty$  corresponds to the value of the diagonal permittivity elements for infinite frequency. With that, we arrive at

$$\varepsilon(\omega) = \mathbb{1}\varepsilon_\infty + \omega_q^2 M(\omega)^{-1}. \quad (50)$$

In the anisotropic case, i.e. when the stiffness and damping coefficients of all oscillators are different, the inverse of  $M(\omega)$  and thus also the permittivity tensor  $\varepsilon(\omega)$  exhibit a very complicated  $\omega$  dependence. However, many magneto-optical materials are isotropic, that is,  $\Omega_x = \Omega_y = \Omega_z = \Omega$  and  $\Gamma_x = \Gamma_y = \Gamma_z = \Gamma$ . In this case we obtain

$$\varepsilon = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{21} & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix} \quad (51)$$

with

$$\varepsilon_{11} = \varepsilon_{22} = \frac{\omega_q^2(-\omega^2 + \Omega^2 - 2i\Gamma\omega)}{(-\omega^2 + \Omega^2 - 2i\Gamma\omega)^2 - \omega^2\omega_c^2} + \varepsilon_\infty \quad (52a)$$

$$\varepsilon_{12} = -\varepsilon_{21} = \frac{-i\omega\omega_c\omega_q^2}{(-\omega^2 + \Omega^2 - 2i\Gamma\omega)^2 - \omega^2\omega_c^2} \quad (52b)$$

$$\varepsilon_{33} = \frac{\omega_q^2}{-\omega^2 + \Omega^2 - 2i\Gamma\omega} + \varepsilon_\infty. \quad (52c)$$

From the equations (52) we can see that the magnetic field not only influences the off-diagonal elements of the dielectric function but also the diagonal elements, which can be regarded as the classical manifestation of the *Zeeman effect* [51]: In the case of low damping ( $\Gamma$  is small), the resonance condition for  $\varepsilon_{11}$  is given by

$$\omega \approx \Omega \pm \frac{\omega_c}{2}, \quad (53)$$

which means that the material resonance is split by the magnetic field.

However, in most magneto-optical applications, the influence of the magnetic field on the diagonal components can be neglected, since usually  $|\omega_c| \ll \omega, \Omega, \Gamma$ . In this approximation the components of  $\varepsilon$  reduce to

$$\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \frac{\omega_q^2}{-\omega^2 + \Omega^2 - 2i\Gamma\omega} + \varepsilon_\infty \quad (54a)$$

$$\varepsilon_{12} = -\varepsilon_{21} = \frac{-i\omega\omega_c\omega_q^2}{(-\omega^2 + \Omega^2 - 2i\Gamma\omega)^2}. \quad (54b)$$

The model function (54) provides a very good qualitative description of the magneto-optic response of many materials. There are several important properties of the model: firstly, since  $\omega_c$  is proportional to the magnetic field, the model correctly predicts a linear relation between the off-diagonal elements of the permittivity and the magnetic field. In the regime of weak magnetic fields, this is the case for most magneto-optic materials. Furthermore, the model provides a Kramers–Kronig-consistent relation between the real and imaginary part of  $\varepsilon_{12}$  [44].

The permittivity function can be easily implemented in many numerical simulation tools. However, to obtain quantitatively correct simulation results, the free model parameters in the relations (54) have to be fitted to measurement data. For

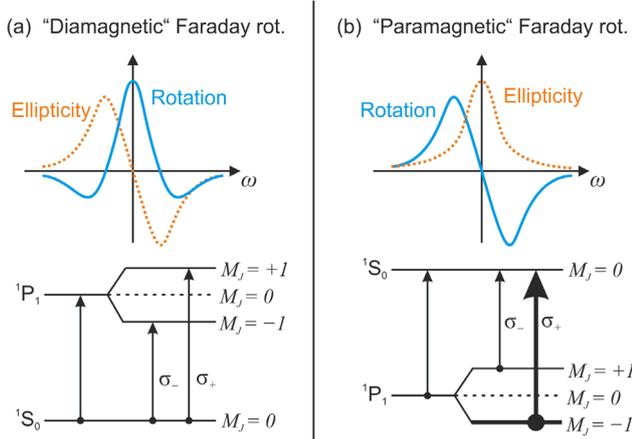
some MO materials it can be necessary to add further oscillators to the polarization (46) with different individual parameter sets, in order to account for a more complex MO behavior.

**2.3.2. Quantum mechanical description.** Compared to the simple mechanical model provided in section 2.3.1 the advantage of the following quantum mechanical description is that it can explain different dispersion behaviors of the magneto-optic spectra (i.e. diamagnetic and paramagnetic lineshapes) as well as the influence of temperature on the magnitude of the Faraday rotation. The following discussion is derived from [44] and [53].

To understand the origin of Faraday rotation in atomic systems, we will make use of the fact that Faraday rotation can be regarded as a manifestation of magnetic circular dichroism (MCD), i.e. different absorption of right-handed circularly polarized (RCP) and left-handed circularly polarized (LCP) light [44]. The reason is that it is much simpler to relate atomic properties to MCD than to the Faraday effect directly. We note in passing, that, as in the case of optical activity [6, 54], polarization rotation and circular dichroism are connected via Kramers–Kronig relations [44].

In magneto-optics it is common to distinguish two contributions to MCD: firstly, due to the Zeeman effect, atomic levels are split into several levels that can be excited only with light of the correct handedness. This results in absorption maxima of LCP and RCP light which are slightly split in energy. As a consequence, the absorption of LCP and RCP is different for most wavelengths. This mechanism is referred to as *diamagnetic* Faraday rotation, as it often occurs in diamagnetic materials. The second mechanism contributing to MCD (if present) usually exceeds the diamagnetic contribution: due to temperature or other influences, the effective oscillator strength of either the LCP or RCP transition can be suppressed. This can dramatically increase the absorption difference of LCP and RCP and thus the Faraday rotation. This mechanism is called *paramagnetic* Faraday rotation as paramagnetic materials often exhibit such a behavior. At this point it should be emphasized that the terms diamagnetic and paramagnetic are not related to the magnetic susceptibility but are part of an established naming scheme for classifying the dispersion in magneto-optical spectra [44].

A more detailed comparison between the diamagnetic and paramagnetic mechanisms is illustrated in the figures 5(a) and (b) respectively. In the diamagnetic case the ellipticity spectrum is S-shaped (orange line) whereas the Faraday rotation shows a peak surrounded by two smaller negative peaks (blue line). This behavior is very similar to chiral media [54]. The lower part of figure 5(a) shows an exemplary and idealized energy level scheme which would result in such a magneto-optic dispersion. In this example we consider a transition from a  $^1S_0$  state to a  $^1P_1$  state. Due to the magnetic field the  $^1P_1$  state is split into three states with quantum numbers  $M_J = 0, +1, -1$ . An incoming LCP or RCP photon induces a transition with  $\Delta M = +1$  or  $\Delta M = -1$ , respectively. Due to the magnetic splitting, the absorption lines of the LCP and RCP transitions (denoted by  $\sigma_+$  and  $\sigma_-$ ) are shifted in energy. Thus, the absorption for LCP and RCP light is different for



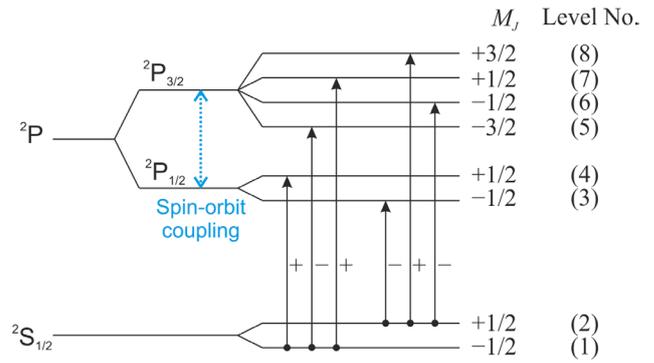
**Figure 5.** Comparison of the diamagnetic (a) and paramagnetic (b) Faraday rotation mechanisms. The line shapes of Faraday rotation and ellipticity are drawn schematically. In (b), the  $M_J = -1$  level is more densely populated, indicated by a thicker line.

most wavelengths and results in the depicted Faraday rotation dispersion. We note that this is in fact the dispersion behavior as predicted by the mechanical oscillator description in section 2.3.1.

A different behavior can be observed in the paramagnetic case, which is schematically depicted in figure 5(b). This situation arises when the oscillator strengths of the LCP and RCP transitions is significantly different. To illustrate this, in the lower part of figure 5(b) an idealized energy level diagram is depicted. The transition energies and the magnetic field induced splitting are assumed to be the same as in the diamagnetic example. Hence, there should also be a diamagnetic contribution to the Faraday rotation spectrum with the same magnitude. However, here the usually much stronger paramagnetic mechanism dominates the magneto-optical spectra: For  $T \rightarrow 0$  K the population probability of the ( $M_J = -1$ ) level is much higher than for the ( $M_J = +1$ ) level, resulting in a magnetization. Therefore, the  $\sigma_+$  transition dominates. This difference in oscillator strength is responsible for the paramagnetic Faraday rotation dispersion.

The absolute magnitude of the magneto-optical response depends on the specific atomic structure of the atoms in the medium. In the case of solids, the resulting band structure is critical. There are many factors that contribute to a strong magneto-optic response and also to a dispersion behavior deviating from the two idealized cases discussed above. This includes spin-orbit interactions, exchange interactions, and further effects. A more detailed compilation of possible influences is left to specialized literature, such as [44, 53, 55, 56]. However, in the following it is motivated why spin-orbit interaction is one of the most important ingredients for a large Faraday rotation response (as is the case for Eu compounds).

To understand the influence of spin-orbit coupling, we consider the exemplary dipole transitions depicted in figure 6. The diagram shows transitions from a  $^2S_{1/2}$  state to  $^2P$  states. Due to the spin-orbit interaction the  $^2P$  state is split into the two levels  $^2P_{1/2}$  ( $J = 1/2$ ) and  $^2P_{3/2}$  ( $J = 3/2$ ). The presence of a magnetic field lifts the degeneracy of levels with the same total



**Figure 6.** Schematic diagram of electric dipole transitions to illustrate that a large spin-orbit coupling can contribute to strong Faraday rotation.

angular momentum. The arrows indicate the dipole allowed transitions which fulfill the selection rule  $\Delta M_J = \pm 1$ . It can be shown [44] that the non-zero transition dipole moments  $d_{ab}^{\pm}$  fulfill the relations

$$|d_{14}^+|^2 = |d_{23}^-|^2 = (2/9)d^2 \quad (55)$$

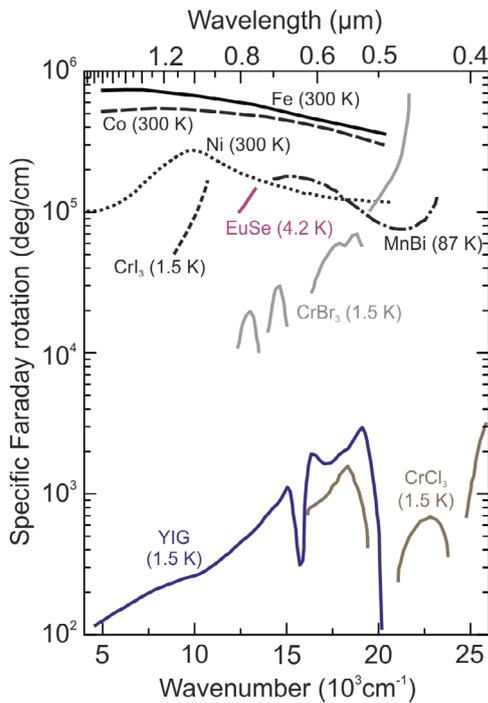
$$|d_{15}^-|^2 = |d_{28}^+|^2 = (1/3)d^2 \quad (56)$$

$$|d_{17}^+|^2 = |d_{26}^-|^2 = (1/9)d^2, \quad (57)$$

where  $d$  is constant. For  $T \rightarrow 0$  only level 1 is populated and only the transitions to the levels 4, 5 and 7 are possible. Neglecting the difference in transition frequency, their contribution to MCD (and thus Faraday rotation) would cancel out, since from equation (55) follows  $|d_{14}^+|^2 - |d_{15}^-|^2 + |d_{17}^+|^2 = 0$ . This means that the Faraday rotation occurs only due to the different transition frequencies, as it was the case in the diamagnetic mechanism discussed above. It can be seen directly from figure 6 that, with increasing spin-orbit splitting, the difference in the transition frequencies become larger and thus the overall Faraday rotation becomes larger. When the temperature increases, the population of level 2 starts to grow and also contributes to the Faraday rotation. In the high temperature limit, the populations of level 1 and 2 are the same and it can be shown that the overall contribution to the Faraday rotation by the transitions from level 2 is of the same magnitude as the contribution by the transitions from level 1, yet with opposite sign [44]. More precisely, the Faraday rotation is proportional to the population difference between level 1 and 2, i.e. to the average magnetic moment of the atom or ion (as in the discussion of the paramagnetic mechanism). In summary, we have seen that Faraday rotation tends to decrease with higher temperatures (and lower average magnetic moment) and strong spin-orbit interaction can be beneficial for large rotation angles.

#### 2.4. Comparison of typical magneto-optic materials

Materials with a particularly strong magneto-optic response are usually referred to as magneto-optic (MO) materials. The general properties of these materials differ strongly, not only with respect to the magnitude of the MO response, but also in

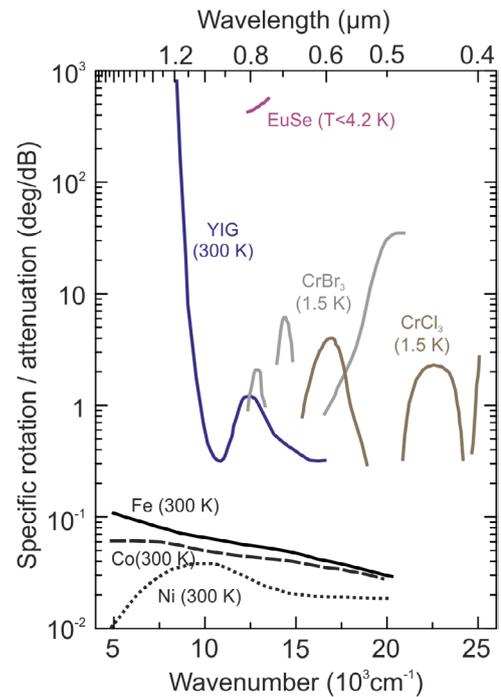


**Figure 7.** Faraday rotation per unit length for different magneto-optic materials. The data were extracted from [55].

many other aspects. For example, there are metallic as well as non-metallic MO materials, MO materials with different magnetic susceptibility, as well as transparent and opaque materials (depending on wavelength). Furthermore, the MO properties usually strongly depend on temperature. In this section, some of the most relevant MO materials are compared. However, it should be mentioned that the collection of materials provided here is not complete, especially because there are many doped variants of the listed materials as well as materials with slightly modified chemical composition. An extended overview can be found in more specialized literature [44, 53, 56–60].

Figure 7 gives a rough overview of the range of available MO materials and the magnitude of the achievable MO response in terms of Faraday rotation per unit length. The data was extracted from [55]. The graph shows that the materials with the largest specific Faraday rotation response are ferromagnetic metals such as Fe, Co, and Ni. Also quite strong Faraday rotation can be achieved by using Europium compounds at low temperatures such as EuSe or by  $\text{CrI}_3$ ,  $\text{CrBr}_3$  and MnBi. EuS, EuTe and EuO are not shown in the diagram but exhibit optical and MO properties similar to EuSe [53, 58]. Relatively weak Faraday rotation is obtained by yttrium–iron–garnet (YIG) as well as by  $\text{CrCl}_3$  at shorter wavelengths. We should note at this point that although the spectrum of YIG is plotted for low temperatures [55] the material also exhibits Faraday rotation of comparable magnitude at room temperature [61].

In many cases the specific Faraday rotation is not a suitable figure of merit for the performance of a MO material. Especially, when a MO material is to be used in transmission geometry, the absorption of the material is relevant. Thus,



**Figure 8.** Faraday rotation normalized to attenuation in dB for different magneto-optic materials. The data were extracted from [55].

a more useful way to characterize MO materials can be the specific Faraday rotation normalized to attenuation (in  $\text{dB cm}^{-1}$ ) shown in figure 8 (data from [55]). This graph reveals that ferromagnetic metals, which have by far the largest MO response, also exhibit the largest optical losses. Although their room temperature compatibility is convenient, their opacity makes them only useful for MO effects in reflection geometry (e.g. for the MO Kerr effect). In contrast, EuSe exhibits both high MO response and high transparency. This is also the case for other Eu compounds such as EuS, EuTe, and EuO, which are not plotted here [53, 58]. Although the ratio of Faraday rotation and attenuation can be a meaningful quantity, it should not be regarded as an universal figure of merit for MO applications. For example, in the near IR YIG exhibits a very high ratio of Faraday rotation and attenuation. However, the absolute Faraday rotation in this frequency region is extremely weak (see figure 7). This means that a polarization rotation device made of YIG would require a relatively large amount of MO material in order to achieve a sizable rotation angle. Hence, for realizing a small scale Faraday rotator, an Eu compound would be more suitable, yet at lower temperatures.

In state of the art nanoscale MO systems, there are three predominant groups of utilized MO materials: metals [21, 30–33], iron garnets [34, 36, 62, 63] and Eu compounds (sections 4 and 6). Due to the high optical losses of metals, the first group is mainly used for systems that operate in reflection geometry (i.e. for utilizing the MO Kerr effect). In the following, we focus on the comparison between iron garnets and Eu compounds, which are suitable for the utilization in Faraday geometry.

Since garnets possess a relatively low Verdet constant, there have been efforts to modify their chemical structure in

order to enhance their MO response. For example, in the case of YIG, substituting yttrium with bismuth leads to a greatly increased Faraday effect [60]. However, the relatively large Faraday rotation of bismuth–iron–garnet (BIG) comes with significant fabrication difficulties: BIG only forms in sophisticated non-equilibrium processes, which involve pulsed laser deposition as well as a subsequent high temperature annealing. This makes the fabrication of hybrid systems incorporating plasmonic nanostructures very challenging and restricts the number of possible structure geometries considerably. For example, any gold nanostructures that are incorporated inside a BIG film would get damaged due to the high temperatures. Moreover, BIG only grows on garnet substrates or special buffer layers. However, solutions to overcome these problems such as coating with a few nm of Al<sub>2</sub>O<sub>3</sub> or SiO<sub>2</sub> by evaporation or atomic layer deposition might be possible [64, 65].

Eu compounds such as EuSe and EuS provide very simple fabrication by physical vapor deposition and are also compatible with standard electron beam lithography processes. As such, these materials can be used to realize sophisticated layer based nanostructure geometries consisting of both magneto-optic and plasmonic materials. EuSe and EuS possess very strong MO response below and around their Curie temperatures at 7 K and 16.6 K, respectively.

For the sake of completeness, it should be mentioned that outside of the visible and near infrared regime there is a higher availability of materials with both high Verdet constant and reasonable transparency. For example, in 2011 Shuvaev *et al* showed that in the THz regime HgTe can provide 14° Faraday rotation already for a 70 nm thick film at 1 T [66]. Furthermore, Crassee *et al* demonstrated in 2010 that at low temperatures a single layer of graphene can produce Faraday rotation on the order of 5° in the THz regime for a magnetic field of 7 T [67].

## 2.5. Plasmonics

In this section the basic optical properties of metal interfaces and nanostructures are summarized. This allows us to understand the emergence of localized surface plasmon resonances in gold nanostructures as used in the following sections. It will turn that plasmonic resonances are of fundamental importance for tailoring the dispersion properties of hybrid magnetoplasmonic systems.

This section begins with a discussion of the general optical properties of metals based on the plasma model (also known as Drude model). This is the foundation for the subsequent analysis of the two most prominent plasmonic phenomena: surface plasmon polaritons and localized surface plasmon resonances. As this section cannot provide full coverage of all aspects of plasmonics and related electrodynamic effects, the interested reader can find very detailed further discussions in the works [46, 68], which this section is based on.

**2.5.1. The dielectric function of metals.** The characterizing optical properties of metals can be derived from the *plasma model*, also known as the *Drude model*. For many metals

the validity of this simple model extends over a surprisingly wide wavelength range. The basic assumption of the model is that the unbound electrons with number density  $n$  are moving freely against the fixed and positively charged ions. This means that the electrons are not subjected to a restoring force as in the Lorentz oscillator model discussed in section 2.3.1. However, as in the Lorentz model, the electrons are assumed to be driven by the time harmonic electric field  $\mathbf{E}(t) = \mathbf{E}_0 e^{-i\omega t}$  of the light wave. The resulting equation of motion for the electron displacement  $\mathbf{x}(t)$  is given by

$$m\ddot{\mathbf{x}}(t) + m\gamma\dot{\mathbf{x}}(t) = -e_0\mathbf{E}_0 e^{-i\omega t}, \quad (58)$$

where  $m$  and  $-e_0$  are the effective mass and charge of an individual electron. Furthermore, the damping constant  $\gamma = 1/\tau$  corresponds to the characteristic collision frequency of a plasma electron. The differential equation (58) can be solved by the time harmonic ansatz

$$\mathbf{x}(t) = \mathbf{x}_0 e^{-i\omega t}, \quad (59)$$

which leads to the solution

$$\mathbf{x}(t) = \frac{e_0}{m(\omega^2 + i\gamma\omega)} \mathbf{E}(t). \quad (60)$$

Furthermore, by means of equation (60), the macroscopic polarization of the plasma  $\mathbf{P} = -ne_0\mathbf{x}$  can be written as

$$\mathbf{P} = \frac{-ne_0^2}{m(\omega^2 + i\gamma\omega)} \mathbf{E}. \quad (61)$$

With equation (61) we have now established the relation between the polarization of the plasma and the applied electric field. Hence, via the relation  $\mathbf{P} = \varepsilon_0\chi(\omega)\mathbf{E}$ , the electric susceptibility  $\chi(\omega)$  can be identified as

$$\chi(\omega) = \frac{-ne_0^2}{m\varepsilon_0(\omega^2 + i\gamma\omega)}. \quad (62)$$

Finally, by introducing the *plasma frequency*  $\omega_p$

$$\omega_p = \frac{ne_0^2}{m\varepsilon_0} \quad (63)$$

and by using  $\varepsilon(\omega) = 1 - \chi(\omega)$ , the complex dielectric function of the free electron gas can be written as

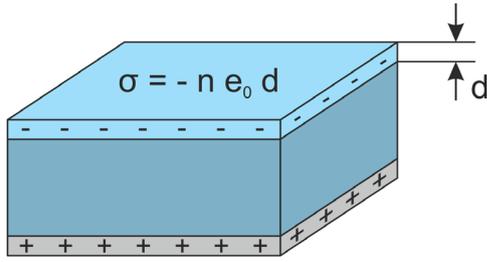
$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}. \quad (64)$$

The real and imaginary parts of  $\varepsilon(\omega)$  are given by

$$\text{Re}[\varepsilon(\omega)] = 1 - \frac{\omega_p^2\tau^2}{1 + \omega^2\tau^2} \quad (65)$$

$$\text{Im}[\varepsilon(\omega)] = \frac{\omega_p^2\tau}{\omega + \omega^3\tau^2}. \quad (66)$$

In order to increase the agreement between the modeled permittivity (64) and the one of real metals, it can be useful to introduce a slight modification. By adding a constant parameter to the expression (64), the contribution of the positive metal ions to the overall polarization can be taken into account [68]. With this modification, the dielectric function becomes



**Figure 9.** Illustration of a volume plasmon. The electrons in the metal are collectively driven at the plasma frequency  $\omega_p$ . They are displaced by the distance  $d$  relative to positively charged ions. This results in a surface charge density  $\sigma = -ne_0d$ .

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \quad (67)$$

where  $\varepsilon_\infty$  is the permittivity at infinite frequency. Before the behavior of this model function is discussed in further detail, the role of the plasma frequency  $\omega_p$  should be clarified.

In order to understand the physical meaning of the plasma frequency  $\omega_p$ , we now consider a plane wave propagating in an electron plasma with the dielectric function (64). For the sake of simplicity any material losses are neglected, i.e. we assume  $\gamma = 0$  and obtain

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \quad (68)$$

For the propagating electromagnetic wave we make the ansatz

$$\mathbf{E}(z, t) = \mathbf{E}_0 e^{i(kz - \omega t)}. \quad (69)$$

By inserting the equations (68) and (69) into the Helmholtz wave equation (18) the dispersion relation

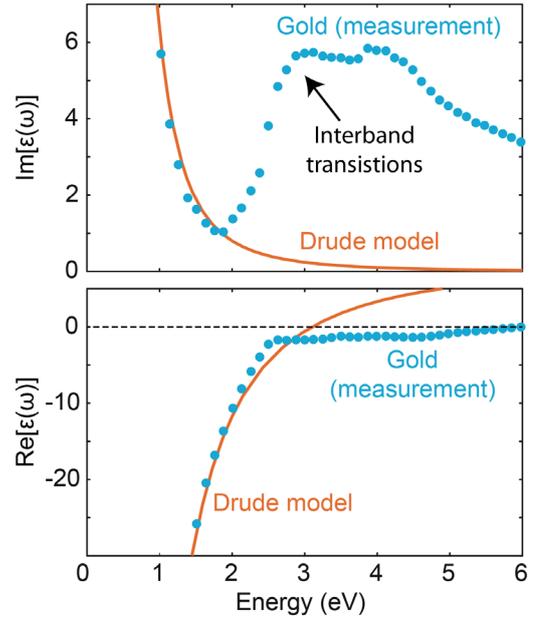
$$k^2 = \frac{\omega^2}{c_0^2} \varepsilon(\omega) = \frac{\omega^2 - \omega_p^2}{c_0^2} \quad (70)$$

is obtained. It is evident that there are two distinct frequency regimes: for  $\omega < \omega_p$  the propagation constant  $k$  becomes imaginary, i.e. there is no light propagation possible. On the other hand, for  $\omega > \omega_p$  transverse electromagnetic waves can propagate through the plasma, as  $k$  is real.

A physically interesting situation occurs if  $\omega = \omega_p$ . In this case, the propagation constant  $k$  vanishes, i.e. the electron excitation becomes a collective movement. Furthermore, the dielectric function  $\varepsilon(\omega_p)$  becomes zero. Together with  $\mathbf{D} = \varepsilon_0 \varepsilon(\omega_p) \mathbf{E} = 0$  the equation (14) yields

$$\mathbf{E} = -\mathbf{P}/\varepsilon_0, \quad (71)$$

which means that the electric field is purely originating from the polarization of the medium. Figure 9 illustrates how this situation can be interpreted in the case of a flat piece of metal. The electrons are collectively displaced by a distance  $d$  leading to the surface charges  $\sigma = -ne_0d$  and  $\sigma = +ne_0d$  on the two sides of the slab. The attractive force between the two charged surfaces normalized to a single electron can be written as  $F = m\ddot{d} = -e_0|E|$  with  $|E| = ne_0d/\varepsilon_0$ . This results in the equation of motion



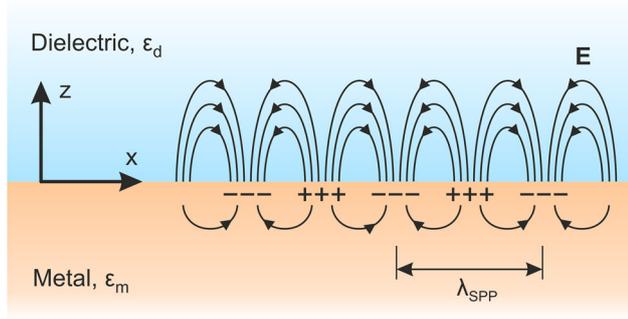
**Figure 10.** Comparison of the measured dielectric function of gold (blue dots) and the fitted model function (67) (orange curves). For low frequencies there is a good agreement between the modeled permittivity and the measurement. However, for higher frequencies interband transitions cause a significant deviation from the model. The measurement data was extracted from [68].

$$\ddot{d}(t) + \omega_p^2 d(t) = 0. \quad (72)$$

From this follows that the plasma frequency can be interpreted as the characteristic frequency of a collective electron gas that is oscillating relatively to a fixed positively charged background. The excitation of such an oscillation is referred to as *volume plasmon*. At this point it should be noted that volume plasmons are a fundamentally different type of excitation compared to surface plasmons and localized surface plasmons, which is discussed in the next sections. Volume plasmons are of longitudinal nature and can thus not couple to transverse light waves. Thus, they do not play a role for the magnetoplasmonic systems discussed in the following sections.

Figure 10 depicts a comparison of the Drude model function (67) (orange curves) fitted to the measured permittivity of the most popular plasmonic material, namely gold (blue dots). The data was extracted from [68]. The top panel shows the imaginary part, whereas the lower panel displays the real part of  $\varepsilon(\omega)$ . For photon energies below 2 eV (above 650 nm), there is good agreement between the model and the measurement. However, for larger frequencies the interband transitions of gold cause an increase of  $\text{Im}(\varepsilon)$ . Silver exhibits a similar behavior (not shown here), however the accuracy of the Drude model extends until electron energies of about 3.5 eV (350 nm). Also for many other metals the Drude model yields a very accurate description of the permittivity, especially in the infrared.

In principle, the Drude model can be extended to take the interband transitions into account by adding further oscillators that contribute to  $\varepsilon(\omega)$ . In contrast to the equations of motion (58), these additional oscillators are then subjected



**Figure 11.** Illustration of a surface plasmon polariton (SPP) propagating along a flat metal-dielectric interface at  $z = 0$ .

to a restoring force in analogy to the Lorentz oscillators discussed in section 2.3.1.

**2.5.2. Surface plasmon polaritons.** Surface plasmon polaritons (SPPs) are electromagnetic waves propagating along metal-dielectric interfaces. Although SPPs are not directly utilized in the magnetoplasmonic systems analyzed in later sections, the discussion of their most important properties provides an important context for the analysis of localized surface plasmon resonances presented in the next section.

In the following, the electrodynamics of a SPP are analyzed for the simplest case of a flat metal-dielectric interface at  $z = 0$ , as illustrated in figure 11. For an electromagnetic wave propagating in  $x$  direction we make the ansatz

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(z)e^{i(\beta x - \omega t)} \quad (73a)$$

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(z)e^{i(\beta x - \omega t)}. \quad (73b)$$

Since the geometry is invariant in  $y$  direction,  $\mathbf{H}(z)$  and  $\mathbf{E}(z)$  can be assumed to be independent of  $y$ . By inserting (73) into Maxwell's equations for time harmonic fields (16) it can be shown that there are two classes of solutions: TE-polarized waves, with only the components  $H_x$ ,  $H_z$  and  $E_y$  being non-zero and TM-polarized waves, with only the components  $E_x$ ,  $E_z$  and  $H_y$  being non-zero. However, it should be noted that it can be shown that TE waves do not fulfill the condition that  $E_x$  and  $D_z$  are continuous at the metal-dielectric interface [68]. Hence, SPPs are always TM polarized. In the case of the TM waves, the Maxwell equations (16) yield

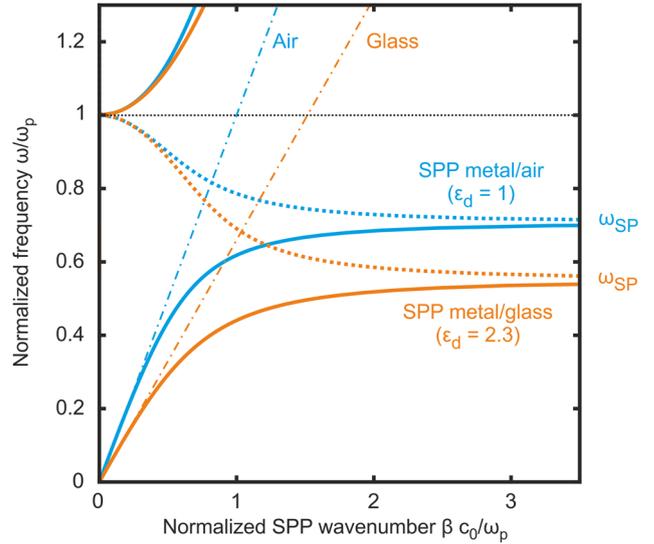
$$\frac{\partial^2}{\partial z^2} H_y(z) + \left[ \frac{\omega^2}{c_0^2} \epsilon(\omega, z) - \beta^2 \right] H_y(z) = 0 \quad (74a)$$

$$E_x(z) = \frac{1}{i\omega\epsilon_0\epsilon(\omega, z)} \frac{\partial H_y(z)}{\partial z} \quad (74b)$$

$$E_z(z) = \frac{-\beta}{\omega\epsilon_0\epsilon(\omega, z)} H_y(z). \quad (74c)$$

Within the dielectric half-space ( $z > 0$ ) the solutions of the equations (74) are given by

$$H_y(z) = C_d e^{-\kappa_d z} \quad (75a)$$



**Figure 12.** Dispersion diagram for surface plasmon polaritons at a metal-air interface (blue) and a metal-glass interface (orange). Solid lines correspond to the real part of the wavenumber  $\beta$ , whereas the dotted lines denote the imaginary part. The dash-dotted lines indicate the corresponding light cones of the dielectric half-spaces.

$$E_x(z) = C_d e^{-\kappa_d z} \frac{i\kappa_d}{\omega\epsilon_0\epsilon_d} \quad (75b)$$

$$E_z(z) = C_d e^{-\kappa_d z} \frac{\beta}{\omega\epsilon_0\epsilon_d} (-1), \quad (75c)$$

where the positive decay constant  $\kappa_d$  has to obey

$$\kappa_d^2 = \beta^2 - \frac{\omega^2}{c_0^2} \epsilon_d. \quad (76)$$

Similarly, for  $z < 0$  the solutions are given by

$$H_y(z) = C_m e^{+\kappa_m z} \quad (77a)$$

$$E_x(z) = C_m e^{+\kappa_m z} \frac{i\kappa_m}{\omega\epsilon_0\epsilon_m} (-1) \quad (77b)$$

$$E_z(z) = C_m e^{+\kappa_m z} \frac{\beta}{\omega\epsilon_0\epsilon_m} (-1) \quad (77c)$$

with the condition

$$\kappa_m^2 = \beta^2 - \frac{\omega^2}{c_0^2} \epsilon_m \quad (78)$$

for decay constant  $\kappa_m$ , which is also positive. At the interface between the two half-spaces the normal component of  $\mathbf{D} = \epsilon_0\epsilon\mathbf{E}$  and the tangential component of  $\mathbf{E}$  have to be continuous. From this follows that  $C_m = C_d$  and

$$\frac{\kappa_m}{\kappa_d} = -\frac{\epsilon_m}{\epsilon_d}. \quad (79)$$

Equation (79) is an important intermediate result. We see that modal confinement ( $\kappa_m, \kappa_d > 0$ ) requires  $\text{Re}(\epsilon_m) < 0$ , if  $\epsilon_d > 0$ . In other words, SPPs indeed occur only at metal-dielectric interfaces. Furthermore, we can use equation (79) to

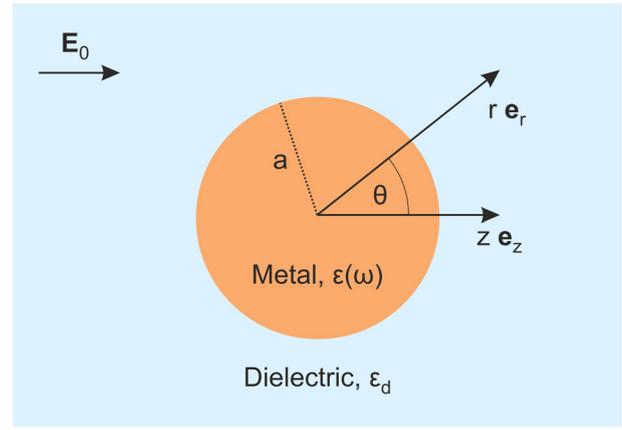
eliminate  $\kappa_m$  and  $\kappa_d$  in the equations (76) and (78) and obtain the dispersion relation of SPPs:

$$\beta(\omega) = \frac{\omega}{c_0} \sqrt{\frac{\varepsilon_m(\omega)\varepsilon_d(\omega)}{\varepsilon_m(\omega) + \varepsilon_d(\omega)}}. \quad (80)$$

For the sake of simplicity, the dispersion of SPPs is discussed for the case of a perfect Drude metal without damping, i.e. it is assumed that the dielectric function of the metal half-space to be (64) with  $\gamma = 0$ . Figure 12 displays the plots of  $\text{Re}(\beta)$  (solid lines) and  $\text{Im}(\beta)$  (dotted lines) for a metal-air interface (blue) and a metal-glass interface (orange). The light cones of the corresponding dielectric half-spaces are indicated by dash-dotted lines. The dielectric function of glass is assumed to be  $\varepsilon_d = 2.3$ . The plots show that the SPP wavenumber is purely imaginary between the plasma frequency and the characteristic surface plasmon frequency  $\omega_{\text{SP}} = \omega_p/\sqrt{1 + \varepsilon_d}$ . Hence, the SPP propagation in this range is prohibited. On the other hand, for  $\omega < \omega_{\text{SP}}$  the SPP wavenumber is purely real and the propagation of SPP waves is possible. For  $\omega > \omega_p$  the metal becomes transparent (see section 2.5.1). Since the dispersion curves of the propagating SPPs lie outside of the light cones of the dielectric materials (dash-dotted lines), these waves cannot be excited by incident plane waves, as the photon energy and momentum cannot be conserved simultaneously. However, there is a variety of techniques that allow to excite SPPs by an incident light beam. Typical examples involve prism couplers, grating couplers and other surface perturbations such as edges or even dust particles. Finally, it should be pointed out that the dielectric function of a real metal does involve significant absorptive contributions, which result in a relatively fast decay of the propagating SPPs. Typical propagation lengths of SPPs are on the order of 10–100  $\mu\text{m}$  [68].

**2.5.3. Localized surface plasmon resonances.** In this section the second important type of plasmonic excitations will be introduced: localized surface plasmon resonances (LSPRs). This plasmonic phenomenon is of fundamental relevance for the magnetoplasmonic structures presented in later sections. In contrast to the surface plasmon polaritons discussed in the previous section, LSPRs are non-propagating excitations. They occur in sub-wavelength metal nanoparticles surrounded by a dielectric medium. Typical resonance wavelengths of gold and silver particles are in the visible and near infrared. The small particle size also results in a high surface curvature, which enables the direct excitation of LSPRs by plane electromagnetic waves. It should be pointed out that LSPRs not only occur in metal volumes which are confined in all three dimensions, but also in structures which are infinitely extended in one dimension (i.e. in metal wires as utilized in the following sections). In that case, the localized oscillation takes place across the wire, perpendicularly to the extended wire axis.

To understand the fundamental mechanism behind a LSPR it is useful to investigate the simple case of a spherical metal particle with a radius  $a$  well below the wavelength of light. This situation is illustrated in figure 13. Due to the small particle size, the electric field of a surrounding electromagnetic wave can be assumed to be constant within a volume of the



**Figure 13.** Illustration of a spherical metal nanoparticle surrounded by a dielectric medium. If the radius  $a$  is well below the wavelength of light, the condition for the localized surface plasmon resonance can be obtained by means of the quasistatic approximation.

size of the sphere. Hence, the electric field of the light wave can be treated as a static field  $\mathbf{E}_0 = E_0 \mathbf{e}_z$  and the system is described by the Laplace equation

$$\Delta \Phi(\mathbf{r}) = 0, \quad (81)$$

where the electric field is obtained via  $\mathbf{E}(\mathbf{r}) = -\nabla \Phi(\mathbf{r})$ . Any time-harmonic dependence can then be added subsequently to the static solution, i.e.  $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{-i\omega t}$ . This assumption is known as *quasistatic approximation*. Since the geometry in figure 13 possesses revolution symmetry around the  $z$ -axis we can write the general solution of (81) as

$$\Phi(r, \theta) = \sum_{l=0}^{\infty} [A_l r^l + B_l r^{-(l+1)}] P_l(\cos \theta), \quad (82)$$

where  $P_l$  are the Legendre polynomials [46]. The coefficients  $A_l$  and  $B_l$  are determined by the following conditions: firstly, at the surface of the metal sphere, the tangential component of  $\mathbf{E}(\mathbf{r})$  and the normal component of  $\mathbf{D}(\mathbf{r}) = \varepsilon_0 \varepsilon(\omega) \mathbf{E}(\mathbf{r})$  have to be continuous. Additionally, it is required that  $\Phi(r, \theta) \rightarrow -E_0 r \cos \theta$  as  $r \rightarrow \infty$ . Applying these conditions results in

$$\Phi(r, \theta) = -\frac{3\varepsilon_d}{\varepsilon + 2\varepsilon_d} E_0 r \cos(\theta) (r < a) \quad (83a)$$

$$\Phi(r, \theta) = -E_0 r \cos(\theta) + \frac{\varepsilon - \varepsilon_d}{\varepsilon + 2\varepsilon_d} a^3 E_0 \frac{\cos(\theta)}{r^2} (r > a). \quad (83b)$$

By comparing the second term in equation (83b) to the generic potential of a dipole in  $z$  direction [46]

$$\Phi(r, \theta) = -\frac{1}{4\pi\varepsilon_0\varepsilon_d} \frac{p \cos(\theta)}{r^2} \quad (84)$$

we can identify

$$\mathbf{p} = 4\pi\varepsilon_0\varepsilon_d \frac{\varepsilon - \varepsilon_d}{\varepsilon + 2\varepsilon_d} a^3 \mathbf{E}_0. \quad (85)$$

Furthermore, by using  $\mathbf{E}_0 = E_0 \mathbf{e}_z$  and  $\mathbf{r} = r \mathbf{e}_r$ , we can express the equations (83) in the form

$$\Phi(\mathbf{r}) = -\frac{3\varepsilon_d}{\varepsilon + 2\varepsilon_d} \mathbf{E}_0 \cdot \mathbf{r} (r < a) \quad (86a)$$

$$\Phi(\mathbf{r}) = -\mathbf{E}_0 \cdot \mathbf{r} + \frac{\varepsilon - \varepsilon_d}{\varepsilon + 2\varepsilon_d} a^3 \frac{\mathbf{p} \cdot \mathbf{r}}{r^3} (r > a) \quad (86b)$$

which is also valid for *any* direction of the electric field  $\mathbf{E}_0$ . So far, we see from equation (85) that the metal particle produces a static dipole field proportional to the applied electric field. Hence, by introducing the polarizability  $\alpha(\omega)$  via the relation  $\mathbf{p} = \varepsilon_0 \varepsilon_d \alpha(\omega) \mathbf{E}_0$ , equation (85) yields the important result

$$\alpha(\omega) = 4\pi a^3 \frac{\varepsilon(\omega) - \varepsilon_d}{\varepsilon(\omega) + 2\varepsilon_d}. \quad (87)$$

We see immediately that when  $\varepsilon(\omega)$  approaches  $-2\varepsilon_d$  the polarizability exhibits a resonant behavior. In the case of an approximately constant  $\text{Im}[\varepsilon(\omega)]$  around that resonance, the resonance condition becomes

$$\text{Re}[\varepsilon(\omega)] = -2\varepsilon_d, \quad (88)$$

which is known as the *Fröhlich condition*. For a nanoparticle consisting of an ideal Drude metal without damping (with a dielectric function (68)) the Fröhlich condition is met at the resonance frequency

$$\omega_{\text{SPR}} = \frac{\omega_p}{\sqrt{1 + 2\varepsilon_d}}. \quad (89)$$

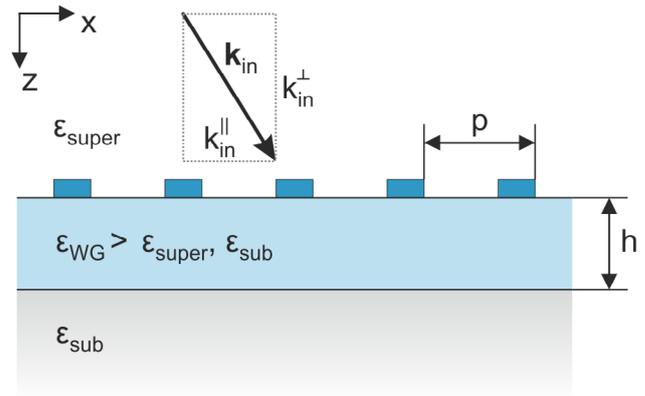
From this we see that an increase of the refractive index around the metal particle causes a red-shift of the resonance. This behavior is utilized in refractive index sensing [69]. With the resonantly increased polarizability (and thus light absorption) also comes a field enhancement inside and in the vicinity of the nanoparticle. This can be immediately recognized by deriving the electric fields  $\mathbf{E} = -\nabla\Phi$  associated with the potentials (86):

$$\mathbf{E}(\mathbf{r}) = \frac{3\varepsilon_d}{\varepsilon + 2\varepsilon_d} \mathbf{E}_0 \quad (r < a) \quad (90a)$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 + \frac{1}{4\pi\varepsilon_0\varepsilon_d} \frac{3(\mathbf{e}_r \cdot \mathbf{p})\mathbf{e}_r - \mathbf{p}}{r^3} \quad (r > a). \quad (90b)$$

The results we have obtained by using the quasistatic approximation are independent of the actual particle size and provide very good accuracy for nanoparticles with sizes below 100 nm. For an increasing particle size the exact derivation of the optical response is still possible for spheres (Gustav Mie 1908 [70]) but it is much more complicated. For most complex particle shapes the derivation of the field distribution and resonance condition needs to be performed numerically. Furthermore, we should note that in the quasistatic approximation radiative losses are neglected. These losses become especially relevant in the limit of low absorption [71] and large particle sizes [68]. One result of the radiative decay is, for example, that even for zero ohmic losses the actual particle polarizability does not fully diverge at resonance.

In summary, as a rule of thumb, for Drude metals as well as for real gold and silver an increase of the particle size in the direction of the light polarization results in a red-shift of the resonance wavelength and a broader absorption line [68].



**Figure 14.** Schematic drawing of a dielectric slab waveguide (light blue) on top of a substrate (grey). The superstrate above the waveguide is drawn as empty white space. The dark blue features indicate a periodic perturbation that is assumed to involve only non-metallic elements. Furthermore, the perturbation is assumed to be so small that the influence on the effective refractive index of the slab waveguide is negligible.

Furthermore, as we will see in section 4, LSPRs also occur in less confined geometries, such as metal wires, if the light polarization is perpendicular to the wire, i.e. in the direction of the metal confinement. In analogy to the case of spheres, a plasmon resonance in a gold wire also red-shifts for increasing wire width.

## 2.6. Resonances in grating-waveguide structures

The Faraday rotation enhancement mechanisms utilized in the sections 4–6 are based on modifying magneto-optic thin films such that they provide both localized surface plasmon resonances, as well as quasiguided waveguide resonances. While the origin of the first resonance type was analyzed in section 2.5, in this section the concept of quasiguided waveguide modes is discussed. The basic dispersion properties of these modes is motivated by first discussing the properties of guided modes and subsequently making the transition to periodically perturbed waveguide slabs via the *empty lattice approximation*. Since this approach is based on highly idealized assumptions, it does not allow to accurately calculate the optical response of the structures introduced in later sections. However, it gives a basic understanding of the most important properties of quasiguided waveguide modes and their origin. This section is based on the discussions in [37–39, 47, 72–74].

Figure 14 shows a schematic drawing of a dielectric slab waveguide. It consists of a thin waveguide (WG) layer with thickness  $h$  and permittivity  $\varepsilon_{\text{WG}} > \varepsilon_{\text{sub}}, \varepsilon_{\text{super}}$  on top of a substrate with permittivity  $\varepsilon_{\text{sub}}$ . The superstrate, i.e. the half-space above the waveguide layer (usually air), is assumed to possess the dielectric function  $\varepsilon_{\text{super}}$ . In order to allow light to be guided inside the center slab, the dielectric functions have to fulfill the condition  $\varepsilon_{\text{WG}} > \varepsilon_{\text{sub}}, \varepsilon_{\text{super}}$ . The entire structure is assumed to be extended infinitely in  $y$ -direction. The dark blue markings indicate a periodic perturbation of the waveguide layer with periodicity  $p$ . For example, this could be a corrugated upper surface of the waveguide film or a dielectric grating with a small extension in  $z$ -direction. In later sections, it is shown that in the

case of a metallic grating, additional plasmonic resonances are introduced to the system. However, in this section only the case of purely dielectric perturbations is considered.

At first, let us consider the situation without any periodic corrugation. In this case the waveguide consists of three flat and homogenous layers. The corresponding optical eigenmodes are obtained by solving the Helmholtz wave equations

$$\Delta \mathbf{E}(\mathbf{r}) + k_0^2 \varepsilon \mathbf{E}(\mathbf{r}) = 0 \quad (91)$$

$$\Delta \mathbf{H}(\mathbf{r}) + k_0^2 \varepsilon \mathbf{H}(\mathbf{r}) = 0. \quad (92)$$

A comparison with (18) shows that the substitution  $k_0 = \omega/c_0$  was used. It can be shown that for the considered geometry there are two types of solutions [47]: transverse electric (TE) modes (with  $\mathbf{E} = E_y \mathbf{e}_y$ ,  $\mathbf{H} = H_x \mathbf{e}_x + H_z \mathbf{e}_z$ ) and transverse magnetic (TM) modes (with  $\mathbf{H} = H_y \mathbf{e}_y$ ,  $\mathbf{E} = E_x \mathbf{e}_x + E_z \mathbf{e}_z$ ). For the TE modes the following ansatz can be made:

$$E_y(x, z) = A e^{a z} e^{i(k_x x - \omega t)} \quad (\text{superstrate}) \quad (93)$$

$$E_y(x, z) = (B e^{i k_z z} + C e^{-i k_z z}) e^{i(k_x x - \omega t)} \quad (\text{WG slab}) \quad (94)$$

$$E_y(x, z) = D e^{-b z} e^{i(k_x x - \omega t)} \quad (\text{substrate}) \quad (95)$$

The ansatz for  $H_y$  in the case of TM modes (not shown here) is very similar. By applying the correct boundary conditions for the material interfaces, the following dispersion relation for TE modes is obtained [47, 74]:

$$\begin{aligned} & h \sqrt{k_0^2 \varepsilon_{\text{WG}} - k_x^2} \\ &= \arctan \left[ \sqrt{\frac{k_0^2 (\varepsilon_{\text{WG}} - \varepsilon_{\text{super}})}{k_0^2 \varepsilon_{\text{WG}} - k_x^2} - 1} \right] \\ &+ \arctan \left[ \sqrt{\frac{k_0^2 (\varepsilon_{\text{WG}} - \varepsilon_{\text{sub}})}{k_0^2 \varepsilon_{\text{WG}} - k_x^2} - 1} \right] + m\pi. \end{aligned} \quad (96)$$

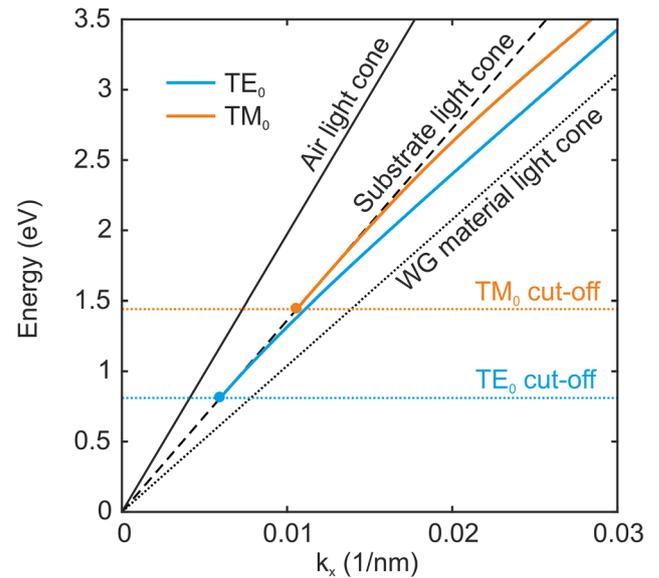
For TM modes the dispersion relation is given by

$$\begin{aligned} & h \sqrt{k_0^2 \varepsilon_{\text{WG}} - k_x^2} \\ &= \arctan \left[ \frac{\varepsilon_{\text{WG}}}{\varepsilon_{\text{super}}} \sqrt{\frac{k_0^2 (\varepsilon_{\text{WG}} - \varepsilon_{\text{super}})}{k_0^2 \varepsilon_{\text{WG}} - k_x^2} - 1} \right] \\ &+ \arctan \left[ \frac{\varepsilon_{\text{WG}}}{\varepsilon_{\text{sub}}} \sqrt{\frac{k_0^2 (\varepsilon_{\text{WG}} - \varepsilon_{\text{sub}})}{k_0^2 \varepsilon_{\text{WG}} - k_x^2} - 1} \right] + m\pi. \end{aligned} \quad (97)$$

The equations (96) and (97) are transcendental equations, which connect the frequency of the light  $\omega = c_0 k_0$  with the propagation constant  $k_x$ . The integer ( $m = 0, 1, 2, 3, \dots$ ) denotes the mode order. For  $\varepsilon_{\text{super}} \neq \varepsilon_{\text{sub}}$  the dispersion relations only possess solutions for photon energies  $E = c_0 \hbar k_0$  above the cut-off energy  $E_{\text{cut}}$ . These cut-off energies are

$$\begin{aligned} E_{\text{cut,TE}} &= \frac{\hbar c_0}{h \sqrt{\varepsilon_{\text{WG}} - \varepsilon_{\text{sub}}}} \\ &\times \left[ \arctan \left( \sqrt{\frac{\varepsilon_{\text{sub}} - \varepsilon_{\text{super}}}{\varepsilon_{\text{WG}} - \varepsilon_{\text{sub}}}} \right) + m\pi \right] \end{aligned} \quad (98)$$

for TE modes and



**Figure 15.** Dispersion of guided modes in a 140 nm thick waveguide. The blue curve corresponds to the TE polarized mode, whereas the orange curve denotes the TM waveguide mode dispersion. The black lines mark the light cones of air, substrate and the waveguide material.

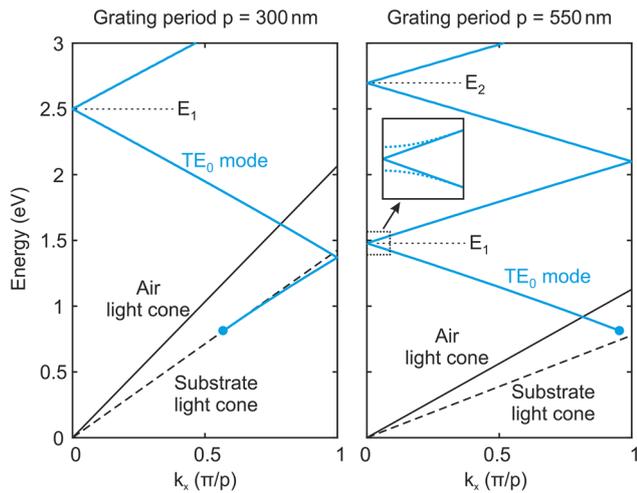
$$\begin{aligned} E_{\text{cut,TM}} &= \frac{\hbar c_0}{h \sqrt{\varepsilon_{\text{WG}} - \varepsilon_{\text{sub}}}} \\ &\times \left[ \arctan \left( \frac{\varepsilon_{\text{sub}}}{\varepsilon_{\text{super}}} \sqrt{\frac{\varepsilon_{\text{sub}} - \varepsilon_{\text{super}}}{\varepsilon_{\text{WG}} - \varepsilon_{\text{sub}}}} \right) + m\pi \right] \end{aligned} \quad (99)$$

for TM modes. Figure 15 displays an exemplary dispersion plot obtained by numerically solving the transcendental equations (96) and (97) for  $m = 0$ . The assumed structure parameters are  $\varepsilon_{\text{WG}} = 3.61$ ,  $\varepsilon_{\text{super}} = 1$ , and  $\varepsilon_{\text{sub}} = 2.1$ , as well as  $h = 140$  nm. The energies of the TE and TM waveguide modes are plotted with blue and orange solid lines, respectively. The corresponding cut-off energies are marked with dotted lines. The light cones of air, waveguide material, and substrate are drawn as black lines. From the plot it can be seen that the modes are always above the light cone of the waveguide material. This means that for a given light frequency, the effective wavelength inside the waveguide slab is always longer than for an infinitely thick waveguide slab (i.e. bulk).

In order to realize the excitation of guided modes by incident light, the following three requirements have to be met: (1) The polarization of the incoming light has to match the polarization of the waveguide modes. This means that TE<sub>0</sub> and TM<sub>0</sub> waveguide modes can only be excited by s-polarized and p-polarized light respectively. (2) Energy conservation requires that the incoming light and the guided light have to have the same frequency  $\omega$ . (3) Momentum conservation requires that the component of the incoming wave vector (see figure 14) matches the wavenumber inside the slab, i.e.

$$k_x = k_{\text{in}}^{\parallel} \quad (100)$$

The latter two conditions implicate that guided modes cannot be excited by incident plane waves: looking at figure 15 reveals that for a given light frequency (energy), the propagation



**Figure 16.** Illustration of the empty lattice approximation for the  $TE_0$  waveguide mode in the slab waveguide structure corresponding to figure 15, but with an additional periodic perturbation.

constant  $k_x$  is always larger than the wavenumber of the light cones of air and the substrate. This means that even for incident light with  $k_{in}^{\parallel} \rightarrow |\mathbf{k}_{in}|$ , the equation (100) cannot be fulfilled.

One way to overcome the problem of simultaneous energy and momentum conservation is by utilizing prism couplers [75, 76]. This technique employs a prism with a refractive index larger than that of the waveguide slab. This prism is brought in contact with the waveguide slab and light is sent through the prism such that it is reflected at the contact interface. Due to photon tunneling, an evanescent wave is then leaking into the waveguide slab. The  $x$  component of this evanescent wave can be matched with  $k_x$  by adjusting the incident angle into the prism.

Another way of coupling the waveguide modes to external plane waves can be realized by introducing periodic perturbations of the waveguide surface, e.g. by attaching a grating on top of the waveguide [47, 73]. In figure 14 such a perturbation is indicated by the dark blue features. If this perturbation is small and the change of the effective refractive index of the waveguide slab is negligible, the modal dispersion can be assumed to be the same as in the non-disturbed case, with the only difference that the range of  $k_x$  can now be reduced to the first Brillouin zone ( $-\pi/p \leq k_x \leq \pi/p$ ) by treating every wavenumber  $k_x$  to be equivalent to  $k_x \pm l2\pi/p$ , with ( $l = 0, 1, 2, 3, \dots$ ) [47]. As a result, the condition (100) relaxes to

$$k_x = k_{in}^{\parallel} \pm l \frac{2\pi}{p}, \quad (l = 0, 1, 2, 3, \dots). \quad (101)$$

This means that the dispersion line is folded back at the borders of the first Brillouin zone ( $k_x = \pm\pi/p$ ). This description is called *empty lattice approximation* [72]. Exemplary results of this approximation are displayed in figure 16. The two plots show the modal dispersion of the same slab waveguide as discussed in figure 15 but with an additional periodic perturbation with  $p = 300$  nm and  $p = 550$  nm. For better readability, only the  $TE_0$  waveguide mode is plotted. The most notable

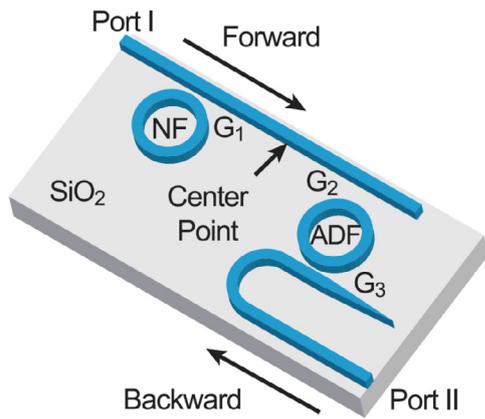
feature of these plots is that as a consequence of the folding of the dispersion line, the  $TE$  modes now reach regions above the air and substrate light cones. In this region the corresponding  $TE$  waveguide modes no longer possess an infinite lifetime and can couple to incoming and outgoing plane waves, i.e. the modes become *quasiguided*. Of special relevance for section 4 are the points where the dispersion lines intersect with the line of  $k_x = 0$ . These points correspond to the case of normal incidence ( $k_{in}^{\parallel} = 0$  and  $l = \pm 1, \pm 2, \dots$ ). For example, in the case of  $p = 300$  nm the first mode accessible for normal incidence has an energy of  $E_1 = 2.5$  eV. On the other hand, in the case of  $p = 550$  nm the first intersection with the energy axis already happens at lower energies such that also the second intersection at  $E_2$  lies within the plotting range. At this point we note the important result that a larger grating period leads to a lower resonance frequency.

Both at the center ( $k_x = 0$ ) and at the edge of the first Brillouin zone ( $k_x = \pi/p$ ) the modes are twofold degenerated due to the line folding. At these points the periodic structure supports two modes with the same energy, but with different symmetry of the electric field distribution with respect to the unit cell of the periodic waveguide. In the simplest case, one of the two modes shows a sinusoidal behavior with respect to the  $x$  direction, whereas the other one shows a cosinusoidal behavior. For normal incidence and systems with high symmetry, usually one of the two eigenmodes is dark, i.e. it possesses a line width of zero, and only the other mode can be excited efficiently. For an increasing incident angle, the line width of the dark mode increases and the mode becomes bright (see also section 4.3). Furthermore, when the periodic corrugation of the waveguide becomes stronger, the degeneracy of the eigenmodes at the center and the edges of the first Brillouin zone is lifted. This is in analogy to the case of electronic Bloch wavefunctions in solids [77] and is indicated by the magnified crop next to  $E_1$  in the right plot of figure 16.

The simple concept of the empty lattice approximation gives a good flavor of the nature of quasiguided modes in corrugated slab waveguides. However, in most real world grating-waveguide systems, the periodic corrugation cannot be treated as a small perturbation anymore. This is also the case for the structures investigated in sections 4–6. Hence, full numerical simulations are required to solve Maxwell's equations in order to accurately predict the modal dispersion and the optical response of such systems. In section 4 we will see that especially the introduction of metallic wire gratings dramatically influences the waveguide dispersion. As the individual grating wires provide an additional localized surface plasmon resonance, which can couple to the quasiguided waveguide modes, a new hybrid excitation emerges: the so-called *waveguide-plasmon-polariton*.

### 3. State of the field

Over the last years the combination of plasmonic and dielectric nanostructures with magneto-optics resulted in a multitude of different nanoscopic systems with intriguing functionalities ranging from enhanced nonreciprocal optical effects to



**Figure 17.** Sketch of the optical isolator based on nonlinear silicon ring resonators as demonstrated by Fan *et al.* From [14]. Reprinted with permission from AAAS.

magnetically controlled optical devices. In this section we provide an overview of magnetoplasmonics and nonreciprocal photonics by systematically grouping the major contributions to the field. To complement our discussion, we also want to make the reader aware of several excellent reviews that focus on different subfields in the realm of plasmonics and magneto-optics [5, 7, 8, 78–85]. Parts of this section have been adapted with permission from the article [40].

### 3.1. Nonreciprocal photonics

The high demand for down-sized optical isolators stimulated a lot of research, which can be divided into two fields: on one hand people studied various ways to enhance the effective Verdet constant of conventional materials. Here, the utilization of plasmonic nanostructures has proven to be a very powerful method as it will be shown in the subsequent sections. On the other hand, there have been realized several nonreciprocal optical systems which do not directly rely on Faraday rotation. These approaches are quite diverse but can be classified with respect to how Lorentz reciprocity is broken.

One way of breaking Lorentz reciprocity is through nonlinear optical effects [7]. This approach is appealing as it allows for passive devices without any external magnetic bias. In particular, for future on-chip environments this aspect is very desirable. For example, much attention was received by the work of Fan *et al* in 2012. They realized a CMOS-compatible all-silicon optical isolator based on ring resonators with a size of  $5 \mu\text{m}$  [14]. The working principle of their two-port device is depicted in figure 17. It relies on the following mechanism: the two ports (silicon waveguides) are coupled via evanescent fields to a nonlinear silicon ring resonator acting as add-drop filter (labeled ADF). Port I is coupled more weakly to the ring resonator than port II. Hence, the amount of energy stored in the ring resonator is different for forward and backward light propagation. As the ring resonator exhibits a strong Kerr nonlinearity, the resonance frequency and thus the frequency of highest transmittance of the system changes depending on the propagation direction. By suitably adding a second ring resonator acting as a notch filter (labeled NF) the overall

system exhibits asymmetric transmission with a transmission ratio of 20 dB. The drawback of this approach is the rather large insertion losses in the order of 30–40 dB. This is also a very common problem among other nonlinear isolators with high transmission ratios [86–88]. On the other hand, nonlinear isolators with low insertion losses often come with rather low transmission ratios [89–91]. However, in a recent theoretical study by Mahmoud *et al* it has been suggested that nonlinear optical isolation could reach practical performance data by utilizing metamaterials [92]. That being said, it has to be stressed that for all passive nonlinear isolators the difference of forward and backward transmittance depends on the light intensity and vanishes for small intensities. This property disqualifies nonlinear isolators for many applications. Furthermore, in a recent study by Shi *et al* it was demonstrated analytically that any optical isolator based on Kerr nonlinearities (as for example presented in the works [14, 87, 93–97]) cannot block a class of low intensity noise in the backward direction when a forward signal is transmitted [7]. They also suspect that this limitation could hold for a much broader range of nonlinear optical isolators.

Another way to break Lorentz reciprocity is spatiotemporal modulation [8]. For example, by electrically [98–105] or mechanically [106–110] modulating the local optical properties of electromagnetic wave carrying structures nonreciprocal operation has been proposed theoretically [98–100, 109–111] and demonstrated experimentally [101–108]. However, while time modulated systems reached practical relevance within the microwave regime, optical realizations are often only theoretical or proof-of-concept realizations [8]. This is because such systems usually come with dramatic losses and it is very difficult to obtain robust modulation with high strength and speed simultaneously [8].

As discussed in the beginning, magnetic fields are the most common way to break Lorentz reciprocity. Besides isolators relying on the Faraday effect there have been demonstrations of nanophotonic isolators based on plasmonic crystals [112] as well as magneto-optic waveguides and ring resonators [13, 113–115]. Among these, the most prominent work is the one by Bi *et al* [13], where a monolithically integrated isolator with 19.5 dB isolation ratio at 1550 nm was experimentally demonstrated. The working principle of this system is based on the effect that the resonance frequency of a silicon ring resonator acting as notch filter depends on light circulation direction. However, as in the case of similar approaches [13, 113–115] there is a dramatic trade-off between isolation ratio and insertion loss. In the case of Bi *et al* the insertion loss is in the order of 20 dB, attenuating the light intensity by a factor of 100.

Furthermore, Davoyan *et al* [116, 117] theoretically proposed a 2D nanophotonic circulator with three ports. Such a nonreciprocal device routes light coming from one port into a specific port depending on the propagation direction. The core element connecting the three junctions consists of plasmonic nanorods embedded inside a magneto-optic bismuth-iron-garnet host. While the fabrication of such a structure might be challenging, Davoyan *et al* demonstrated numerically that under ideal conditions the transmission through one of the

output ports can reach up to 63%, while the transmission of the other (isolated) port is well below 3% [116].

Recently, Scheucher *et al* demonstrated that a circulator can also be realized with a whispering gallery resonator and an attached single rubidium atom [118]. The researchers demonstrated that depending on the quantum state of the atom the optical throughput of two tapered optical fibers attached to the resonator can be routed. The isolation ratios between pairs of the four ports have been demonstrated to be in the order of 10 dB with an insertion loss of around 1.4 dB. Such quantum systems involving so-called chiral coupling have been studied extensively over the last years [119]. Recently, there have been findings in 2D materials that utilize excitons in MoS<sub>2</sub> for optical spin-orbit coupling and spin-momentum locking [120, 121]. These effects might lead also to compact nonreciprocal devices in the future.

An interesting approach towards thin film Faraday isolators is the ansatz by Christofi *et al* [122]. In their recent theoretical study, they utilize BIG nanodisks embedded in silica, which form high-index resonators for the telecom wavelength range. Through careful engineering, they are able to produce a spectral overlap of the electric and magnetic dipole resonances (i.e. the fundamental and higher order Mie resonances). This way it is possible to achieve mostly forward scattering without reflection losses. Such a so-called Huygens metasurface fulfills the Kerker condition [123, 124]. In principle, by utilizing the analog to electromagnetically induced transparency, they can achieve a very high transmission above 90%. Depending on the quality factor of the resonance, Faraday rotations of up to 8° are possible for films of 260 nm thickness at room temperature. To achieve the desired 45° rotation required for a Faraday isolator it might be necessary to stack multiple metasurfaces or to further optimize the quality factor. Furthermore, it would be interesting to see how well experimental values match the theoretical predictions which neglected absorption losses.

At this point we should also note that in addition to all the above mentioned nonreciprocal systems there have also been studies on reciprocal systems with asymmetric light transmission [125]. Such systems mimic some behavior similar to optical isolators but with less restricted symmetry properties [48, 126–130]. However, we stress that optical isolators in the common sense need to break Lorentz reciprocity [5].

### 3.2. Magnetoplasmonic antennas

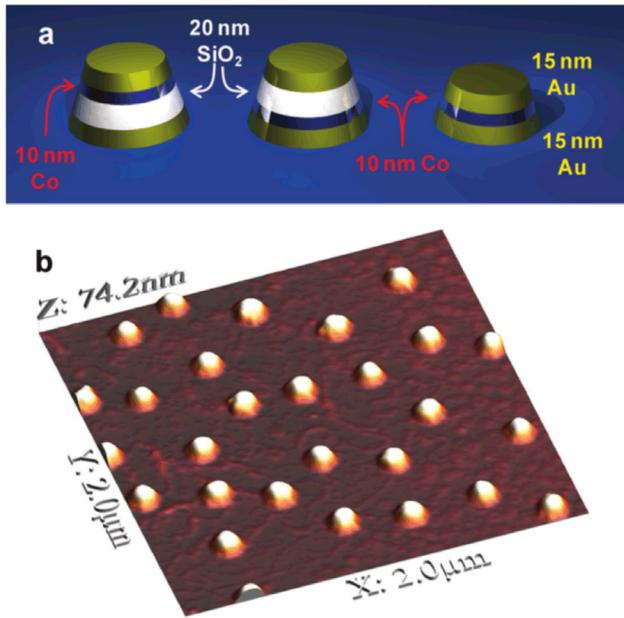
Now we come back to the concept of enhancing magneto-optic (MO) effects of conventional materials by means of plasmonic nanostructures. The most straight-forward way to achieve this is to utilize magnetoplasmonic antennas, i.e. plasmonic nano-antennas which at least partially consist of MO materials. Here, the term antenna refers to any kind of metal nanostructure that provides a localized surface plasmon resonance (LSPR). Possible geometries range from colloidal metal nanoparticles [21] to more well defined systems as for example disks [32] or donuts [131]. It has been demonstrated both experimentally and theoretically that at the LSPR the MO response of magnetoplasmonic antennas is enhanced

[30, 132]. The reason for this behavior is that at the plasmonic resonance not only the diagonal elements of the particle polarizability tensor are resonantly enhanced but also the off-diagonal elements [30, 132], which are the origin of MO activity (see section 2.2.2). For spherical MO particles this behavior can be described analytically via Mie theory [133, 134]. We note in passing that in addition to the resonant plasmonic enhancement there can also be non-resonant contributions to the MO enhancement, which are geometrically induced [135] but usually weaker than the resonant contributions.

While the first theoretical prediction of LSPR-enhanced MO response of plasmonic particles was made in 1987 [136] it was first observed experimentally over 10 years later in granular CoFe-HfO<sub>2</sub> films [137] and in nickel nanowires embedded in alumina [138]. From there on the field of magnetoplasmonics started to emerge. Fabrication techniques such as hole mask lithography and electron beam lithography allowed to study the MO response of geometrically more well defined magnetoplasmonic antennas such as nickel disks [139–141]. As a result, design rules have been derived regarding the disk shape [142] and lattice parameters [33] in order to obtain a tailored and optimized MO response.

While ferromagnetic metals such as cobalt, nickel and iron exhibit very strong MO response, they are also extremely lossy (see section 2.4). This leads to rather broad plasmon resonances in nanoparticles and thus weak MO enhancement. Furthermore, the plasmonic features in the optical spectra are not very well defined. The cleanest plasmon resonances can be obtained from noble metal particles such as gold and silver antennas. Although such nanostructures exhibit rather weak MO activity they have proven to be very useful model systems for in-depth studies on the connections between LSPRs and enhanced MO response. For example, they have been utilized to confirm the accuracy of simple theoretical models for magnetoplasmonic antennas based on discrete dipole approximation [30, 143] and harmonic oscillators [144].

In order to combine high quality plasmon resonances of noble metals with strong MO activity, there have been several intriguing approaches based on hybrid systems. Already very early works on colloidal systems involving both a noble metal and a ferromagnet suggested that the noble metal contribution to the LSPR resonance can lead to a significant MO enhancement [21, 146]. However, as in the case of purely ferromagnetic antennas, by means of lithographic fabrication techniques more control over antenna geometry and composition can be realized. For example, González-Díaz *et al* demonstrated that sandwiches of Co/Au/Co disks fabricated by hole mask lithography exhibit a resonantly enhanced MO Kerr effect around the plasmonic resonance [147]. Furthermore, this enhancement can be tailored and optimized by choosing an appropriate thickness for the chromium and gold layers [31]. Most notably, Banthí *et al* [32] demonstrated that by including a dielectric layer inside a Au/Co/Au sandwich (see figure 18), the electric field distribution of the LSPR mode can be redistributed in such a way that an optimized tradeoff between light absorption and MO response can be achieved. This resulted in a Kerr rotation angle of over 0.3° associated with an extinction of below 0.1 [32].



**Figure 18.** (a) Sketch of the composition of the fabricated nanodisks by Banthi *et al* (b) AFM image of a selected fabricated structure (15 nm Au/10 nm Co/20 nm SiO<sub>2</sub>/15 nm Au). Adapted from [32] John Wiley & Sons. Copyright © 2012 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim.

Interestingly, it is not strictly required that the MO material is part of the resonant antenna itself. It is also possible to enhance the MO response of a continuous (non-resonant) metallic MO film by means of the near field of attached noble metal antennas [148–150]. Similarly, for dielectric MO films it has been demonstrated both theoretically [151] and experimentally [152, 153] that by incorporating noble metal antennas inside or on top of the surface of the MO film the Faraday effect can be enhanced significantly. Baryshev *et al* reported an enhancement of up to 10 times relative to a bare host [153]. However, such systems exhibit extremely low light transmittance in the order of 1% for a 100 nm thick structure. This limits the applicability for photonic devices. To overcome this problem, it was also suggested to compensate for the optical losses by introducing gain by additional pumping [154].

The combination of noble metal antennas with continuous MO films allowed to study several physically interesting coupling effects. For example Torrado *et al* [149] studied magneto-optical effects of localized plasmon resonances interacting with propagating ones resulting in strongly magnetic field dependent SPP dispersion (more discussion on this aspect is provided in section 3.3). Furthermore, Armelles *et al* [150] investigated the interplay between a continuous Au/Co multilayer and a chiral plasmonic oligomer leading to an overlay of chirality induced natural circular dichroism (NCD) and magnetic-field induced circular dichroism (MCD). Similarly, in a very recent study Zubritskaya *et al* [155] demonstrated a metasurface consisting of chiral arrangements of gold and nickel disks which provides magnetically tunable chiroptical response. In general magneto-optic and chiroptic effects can cross-couple and lead to so-called magnetochiral dichroism (MChD), as it was demonstrated by Eslami *et al* for nickel

nano-spirals [156]. This phenomenon describes the situation when the effective permittivity tensor of an optical system exhibits a term proportional to both the wavevector and the magnetic field [156]. While the occurrence of MChD is clearly very interesting its origin is not fully understood yet [157].

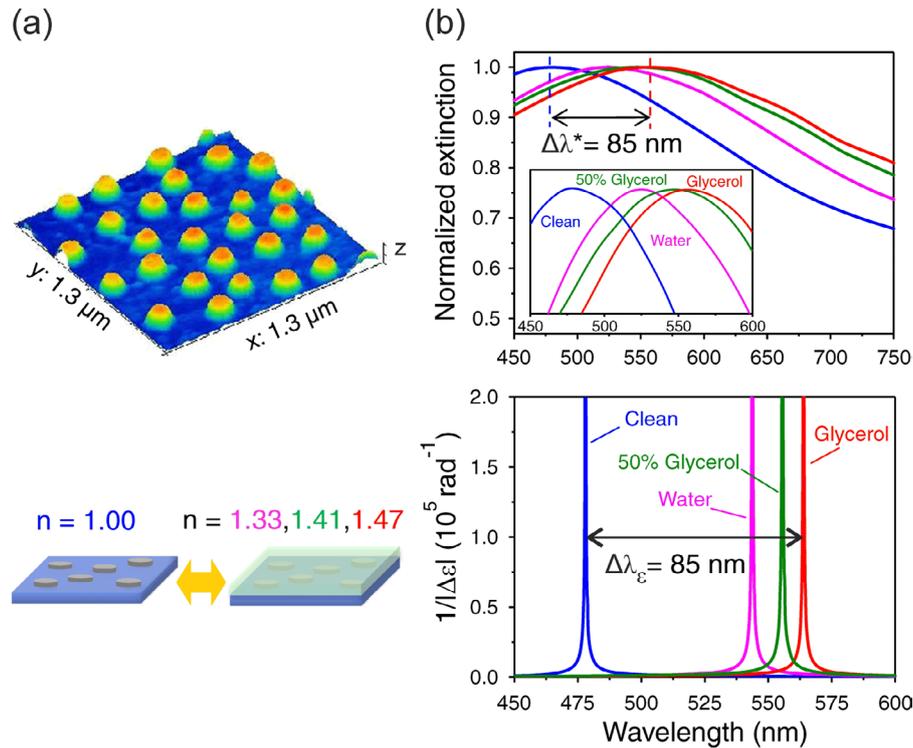
Although the antenna-enhanced MO response in the approaches above is very interesting from a fundamental standpoint, the application in miniaturized nonreciprocal devices, such as Faraday rotators is limited due to either high losses or still too weak MO effects. However, a recent study by Maccaferri *et al* brought up a novel application for magnetoplasmonic antennas, namely plasmonic sensing [145]. Conventional plasmonic sensors rely on tracing a shift of a plasmonic resonance of a nano-antenna resulting from a change of the surrounding refractive index [158] or due to gas exposure [69].

A typical figure of merit for such a sensing mechanism is the ratio of resonance shift for a given environmental change divided by the resonance linewidth in the absorbance spectrum [69]. This means that the resulting measurement accuracy is often limited by the broad linewidth of plasmonic resonances. To obtain significantly increased sensitivity Maccaferri *et al* [145] proposed to not monitor changes of the absorbance spectra but instead to leverage changes in the MO spectra. More precisely, they suggested to monitor the polarization ellipticity variation  $\Delta\varepsilon$  of the light transmitted by an ensemble of Ni metal disks. An AFM image of such an ensemble is depicted in figure 19(a). Figure 19(b) shows a comparison of the extinction spectra (top panel) and the spectra of the quantity  $1/|\Delta\varepsilon|$  (bottom panel) for different surrounding refractive indices. The blue, magenta, green and red curves correspond to air ( $n = 1$ ), water ( $n = 1.33$ ), 50% Vol. glycerol ( $n = 1.41$ ), and glycerol ( $n = 1.47$ ) respectively. Maccaferri and coworkers concluded that the sharp features in the  $1/|\Delta\varepsilon|$  spectra indeed result in a real sensitivity advantage when compared to absorbance based measurements. As such, magnetoplasmonic antennas could form the basis of highly sensitive label-free biosensors.

### 3.3. Propagating surface-plasmon-polaritons in a magneto-optic environment

Besides localized surface plasmon resonances also propagating surface plasmon polaritons (SPP) can exhibit interesting behavior in a magneto-optically active environment. Studies of such systems can be divided into two groups: one one hand it has been investigated how the dispersion behavior of SPPs can be controlled via magnetic fields. The second group of studies concentrated on systems with SPP-enhanced MO effects. In the following, we will give a brief overview of both groups.

SPP are propagating electromagnetic waves at metal-dielectric interfaces (see section 2.5.2). They are widely regarded as a key mechanism for future on-chip optical devices as they allow to guide light through deep sub-wavelength structures [160, 161]. To obtain such devices with high functionality it is important to have the ability to manipulate and switch the SPP propagation at high frequencies. Conventional



**Figure 19.** (a) AFM profile of Ni magnetoplasmonic nanoantennas on glass. (b) Extinction spectra (top panel) and  $1/|\Delta\varepsilon|$  spectra (bottom panel) for different values of the surrounding refractive index. Adapted with permission from Macmillan Publishers Ltd: Nature Communications [145], Copyright (2015).

SPP modulation techniques relied on the refractive index tuning of an active dielectric layer attached to the metal film. However, these approaches either enabled only weak modulation or slow response times [162]. A much more effective way of controlling SPP waves in future photonic devices could be magneto-optic systems. The reason is that the wavevector  $\beta_M$  of a SPP at the surface of a magneto-optically active metal can be modulated via the metal's magnetization  $\mathbf{M}$ . This can be expressed by the relation

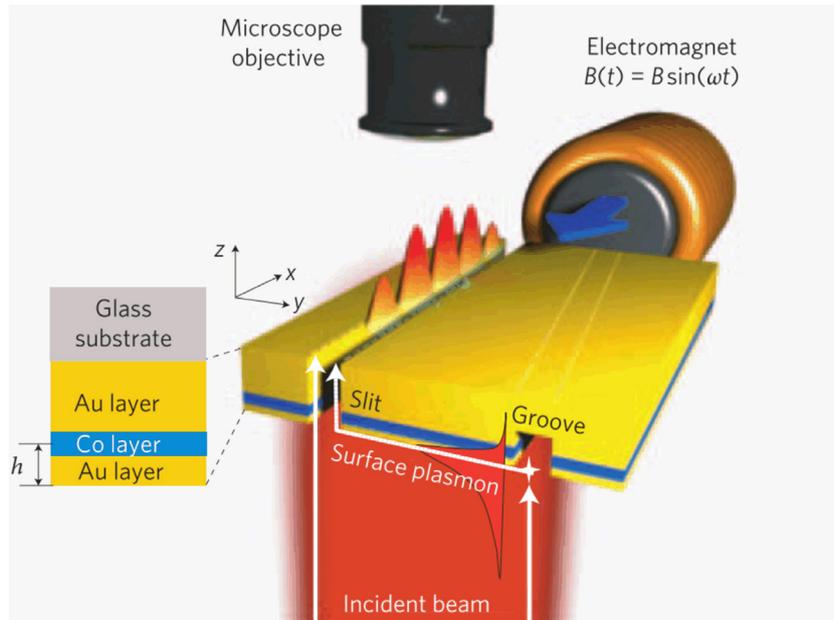
$$\beta_M = \beta + \Delta\beta(\mathbf{M}), \quad (102)$$

where  $\beta$  is the SPP wavevector in equation (80) for zero magnetization of the metal and  $\Delta\beta$  is the magneto-optically induced perturbation [81]. Depending on the direction of  $\mathbf{M}$  relative to the surface and to the wavevector of the SPP,  $\Delta\beta$  is either linear or quadratic in  $\mathbf{M}$ . A comprehensive overview of the different scenarios can be found in the work by Armelles and coworkers [81]. While purely ferromagnetic systems provide the strongest magnetic modulation of SPP waves they also exhibit large ohmic losses. For that reason, similar to the case of magnetoplasmonic antennas, there have been efforts to combine long SPP propagation distances of noble metals with the strong MO response of ferromagnets. Most notably, in 2010 Temnov and coworkers [159] reported magnetically modulated SPP waves in Au/Co/Au films as illustrated in figure 20. They achieved a relative modulation of the SPP wavevector in the order of  $10^{-4}$  for relatively low applied magnetic field strengths of 20 mT enabling potentially high modulation frequencies in the gigahertz regime [162]. In later follow-up work it was

demonstrated that the sensitivity of the SPP dispersion can be further enhanced by an additional dielectric layer [163] and strongly modified by adding resonant plasmonic antennas [149].

Instead of controlling SPP waves magneto-optically, the opposite mechanism, namely utilizing SPP waves to influence MO effects, is also feasible. As already explained in section 2.5.2 the excitation of SPP waves by incident plane waves usually requires special momentum matching techniques. For the first experimental investigations of SPP enhanced MO effects prism couplers were utilized to launch SPP waves on purely ferromagnetic films [164] and later on Co/Au/Co multilayer films [165]. In these works it has been demonstrated that the SPP excitation results in enhanced MO Kerr effect in reflection geometry [164, 165]. Stimulated by these results, in the following years numerous studies have been conducted on optimizing different variations of such geometries [166–170]. It turned out that especially in the case when the magnetization of the film is in plane and perpendicular to the SPP wave vector, the increased MO response can be attributed to the magnetic modulation of the SPP wave vector discussed in the last paragraph [81].

In contrast to continuous films, periodically corrugated systems allow the excitation of SPP resonances and enhanced MO response without a prism coupler [171–177]. This aspect makes such systems much more suitable for the integration in an ultra-compact environment. In this category of magnetoplasmonic systems, especially plasmonic crystals employing the concept of extraordinary transmission [160] added strong momentum to the whole field. For example, in

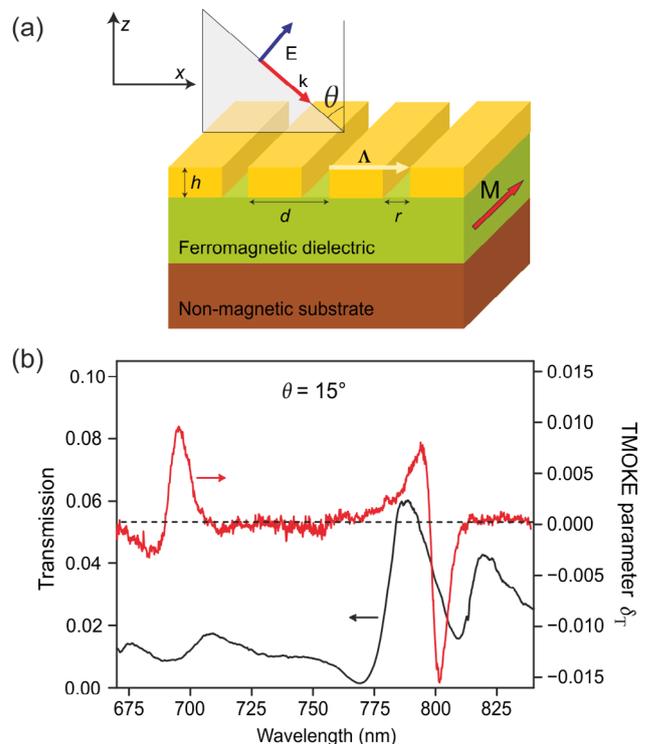


**Figure 20.** Illustration of the SPP modulation technique by Temnov *et al.* A slit-groove pair is milled in a Au/Co/Au multilayer film using a focused ion beam. Surface plasmons are launched by the groove, propagate towards the slit, and interfere with the directly transmitted light to produce a periodic interference pattern along the slit axes. The oscillating magnetic field of an electromagnet is used to periodically switch the magnetization in the thin cobalt layer and thus modify the wave vector of surface plasmons. An imaging nano-optical set-up is used to record the magneto-plasmonic modulation signal. Adapted with permission from Macmillan Publishers Ltd: Nature Photonics [159], Copyright (2010).

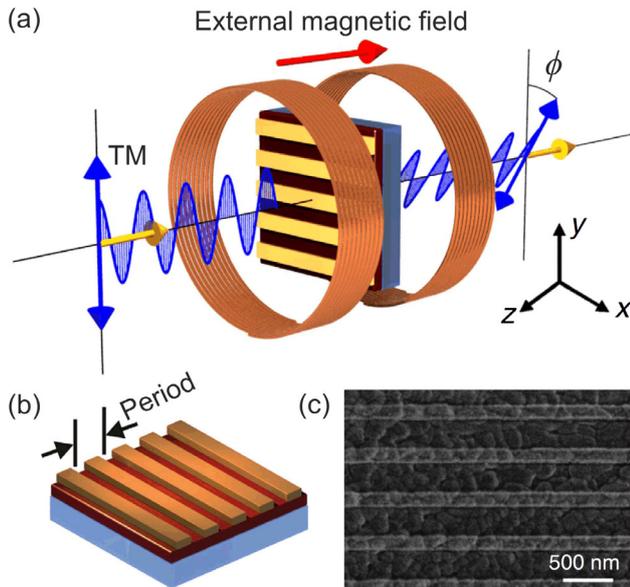
2007 a lot attention was received by the theoretical work by Belotelov and coworkers [63]. They demonstrated by means of rigorous coupled-wave analysis that attaching a 2D gold fishnet structure on top of a dielectric MO active BIG thin film can result in both large Faraday rotation (up to  $0.78^\circ$  for a thickness of less than 200 nm) and also high light transmission of 35% in the near infrared. Responsible for these exceptional performance data is the interaction of quasi-guided waveguide modes (see section 2.6) inside the BIG film with SPP waves in the plasmonic grating [63]. Subsequently, similar theoretical studies were performed on enhanced MO Kerr effect in 1D systems [178, 179]. While so far the particular two-dimensional system from 2007 has not been realized experimentally, in 2011 Belotelov and coworkers demonstrated the experimental realization [62] of the one-dimensional approach. A sketch of the utilized geometry is depicted in figure 21(a). They investigated the enhancement of the TMOKE signal of a dielectric BIG film when a 1D corrugated gold film is deposited on top. As most ferromagnets exhibit strong optical losses TMOKE is usually characterized in reflection geometry (see section 2.2.4). However the high transparency of BIG allowed them to measure in transmission geometry where the TMOKE parameter  $\delta_T$  is defined as relative difference in transmittance for opposite magnetization directions:

$$\delta_T = [T(\mathbf{M}) - T(-\mathbf{M})] / T(0). \quad (103)$$

Belotelov *et al* estimated the TMOKE signal of the bare BIG film to be  $\delta_T \approx 10^{-5}$  and found that by covering the film with



**Figure 21.** (a) Sketch of the magnetoplasmonic crystal for enhanced TMOKE. (b) Measured transmittance and TMOKE parameter  $\delta_T$  for an incident angle of  $15^\circ$ . Adapted with permission from Macmillan Publishers Ltd: Nature Nanotechnology [62], Copyright (2011).



**Figure 22.** (a) Faraday rotation by a magnetoplasmonic crystal for TM-polarized incident light. The Faraday rotation angle is indicated as  $\phi$ . (b) Schematic of the magneto-optical photonic crystal. The BIG film (dark red) is deposited on a glass substrate (blue) and periodic gold nanowires (golden) are sitting atop. (c) Scanning electron microscopy image. Adapted with permission from Macmillan Publishers Ltd: Nature Communications [36], Copyright (2013).

a continuous gold layer the TMOKE signal is increased up to  $\delta_T \approx 10^{-3}$  through the excitation of SPP waves. Most interestingly, their measurement results showed (see figure 21(b)) that by introducing 1D slits into the gold film extraordinary optical transmission can be realized which further boosts the TMOKE signal up to  $\delta_T = 1.5 \times 10^{-2}$  for an incident angle of  $15^\circ$ . Moreover, in 2013 it was demonstrated for a similar geometry that a longitudinal magnetic field can increase the transmittance from 0.04 in the non-magnetized case by up to 24% [180].

These results raised the expectations for future plasmonically driven MO devices. However, being a surface effect, even the plasmonically enhanced TMOKE is still too weak for the most demanding magneto-optical devices such as isolators. As a result, there have been several efforts to enhance the potentially strongest MO effect, namely the Faraday effect [36, 40–42]. Although the investigated geometries in these works might seem quite similar to the ones utilized in the works discussed above, they rely on localized surface plasmon resonances rather than on propagating SPP waves. Hence, they are discussed separately in the following section 3.4. Furthermore, the sections 4–6 provide in-depth discussions of some recent studies that led to the strongest Faraday rotation in thin film geometries so far.

### 3.4. Waveguide-plasmon-polaritons

Through the discussions in the preceding sections, we have identified three main ingredients to realize a good trade-off between strong MO response and low light absorption in magnetoplasmonic systems: the first one is to utilize separate

components responsible for strong MO activity and for pronounced plasmonic resonances [31]. The second ingredient is to decrease the losses by concentrating light inside a transparent dielectric material [32]. The third aspect is to also leverage waveguide mode excitations inside the MO material [62, 63]. All these three aspects are employed in hybrid magnetoplasmonic waveguides as introduced by Chin *et al* in 2013 [36]. Such systems consist of a dielectric MO waveguide with a thickness in the order of 150 nm and an attached noble metal wire grating on top (see figure 22). Unlike in the work by Belotelov *et al* [62, 63, 180] the noble metal grating is much less dense and does not support any propagating SPP waves. Instead, the gold grating has the following two main functions: Firstly, it acts as a waveguide coupler to enable the excitation of quasi-guided waveguide modes inside the MO film for normal incidence. Secondly, it provides a localized surface plasmon resonance that couples strongly to the TM-polarized high-Q waveguide mode inside the MO film leading to the formation of a so-called waveguide-plasmon-polariton (WPP). This quasi-particle was first studied in 2003 in a non-magneto-optic context by Christ and coworkers [37, 38]. However, Chin *et al* discovered that the dispersion properties of a WPP also facilitate strong enhancement of Faraday rotation [36]. In a simple picture this can be understood in the following way: the high oscillator strength of the plasmonic wires enables coupling normally incident light efficiently into the system where it resonates inside the transparent MO film and accumulates Faraday rotation proportional to the Q-factor of the system. In the first realization by Chin *et al* a combination of a 150 nm thick BIG thin film with a 70 nm tick gold grating was utilized. It was demonstrated both experimentally and through numerical simulations that such sub-wavelength thick structures exhibit giant Faraday rotation of up to  $0.8^\circ$  and simultaneously high transmittance of 36% in the near-infrared. Furthermore, in a follow-up study it has been demonstrated that such waveguide-plasmon-polariton systems also show an extremely large TMOKE signal in transmission geometry of up to  $\delta_T = 1.5 \times 10^{-2}$  associated with a transmittance of 45% [34]. While the TMOKE performance of such WPP systems is comparable to the geometry by Belotelov *et al* [62], the obtained Faraday rotation performance was setting a new benchmark at that time.

Starting from there, in the following years there has been a series of works resulting in even thinner Faraday rotators with one order of magnitude larger rotation and more functionality. As the performance of these nanoscopic structures is not far away from meeting the requirements of a Faraday isolator, the concept of hybrid magnetoplasmonic crystals could be a key to realize nonreciprocal photonic devices in a highly integrated environment. The first step in this direction will be discussed in section 4. There we will elaborate on how the hybrid magneto-plasmonic approach by Chin *et al* has been employed and advanced [40] to 220 nm thick devices, which, at low temperatures, show five times greater polarization rotation than in previously reported experiments. In contrast to BIG, the utilized waveguide material EuSe provides a stronger magneto-optical response and also allows much simpler sample fabrication. This aspect is especially crucial

as it opens the way for creating a class of more sophisticated 2D and 3D magnetoplasmonic metamaterials in the future. Furthermore, we discuss how the dispersion properties of such structures can be exploited to freely tailor the working wavelength within the transparency window of the magneto-optical material. In addition, the aspect of active magnetic polarization tuning will be examined.

While the experimental realization and numerical simulation of hybrid magnetoplasmonic systems received considerable attention, until recently, an analytical theoretical description has been missing. In section 5, a simple coupled oscillator model is presented, that reveals the underlying physics inside hybrid magnetoplasmonic systems and yields analytical expressions for the resonantly enhanced magneto-optical response [41]. The Lorentz nonreciprocity of the oscillator model is intrinsically incorporated via the Lorentz force, which is proportional to  $\mathbf{v} \times \mathbf{B}$ . Moreover, the predictions of the model are in good agreement with rigorous numerical solutions of Maxwell's equations for typical sample geometries. The demonstrated ansatz is transferable to other complex and hybrid nano-optical systems and could significantly facilitate device design.

Usually, the maximal MO response of WPP systems is limited by the low  $Q$  factor of the plasmon resonances of the grating. This limitation is lifted elegantly by a novel approach [42] presented in section 6, which is based on a classical optical analog of electromagnetically induced absorption (EIA) [181, 182]. Here, a strongly damped plasmon oscillation is weakly coupled to a narrow linewidth waveguide resonance with a phase delay, leading to constructive interference. By tuning this coupling carefully, a high- $Q$  absorptive hybrid mode is realized, which can be used to resonantly amplify the Faraday rotation response. Furthermore, the EIA mechanism allows to utilize the large oscillator strength of the plasmonic resonance, leading to an efficient coupling of the incident light into the structure without reducing the effective  $Q$  factor due to the broad plasmonic resonance as was the case in previous approaches [34, 36, 40]. Although less than 200 nm thick, at low temperatures, the novel EuS-Au structure design exhibits Faraday rotation of  $14^\circ$ . As this is only a factor of three away from the Faraday isolation requirement, the demonstrated concept could lead to highly integrated, nonreciprocal photonic devices for optical isolation, light modulation, and optical magnetic field sensing.

#### 4. Giant Faraday effect in EuSe-Au structures

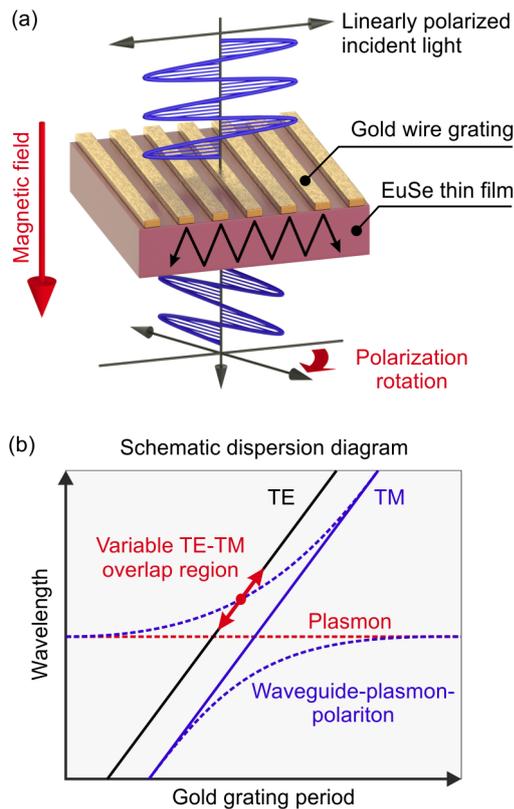
In this section, the realization of an ultra-thin plasmonic Faraday rotator for the visible wavelength regime will be demonstrated. The rotator is a magneto-plasmonic hybrid structure consisting of an EuSe slab and a 1D plasmonic gold grating. At low temperatures, EuSe possesses a large Verdet constant and exhibits Faraday rotation, which does not saturate over a regime of several Tesla. By combining these properties with plasmonic Faraday rotation enhancement, as introduced by Chin *et al* [36], giant Faraday rotation of up to  $4.2^\circ$  for a film thickness of only 220 nm is achieved. This

magneto-optic response is five times stronger than in previously reported experiments. Furthermore, by varying the magnetic field from  $-5$  to  $+5$  T, the polarization of the transmitted light can be continuously tuned over a range of  $8.4^\circ$ . Through experiments and simulations, it will be demonstrated for the first time that the unique dispersion properties of such a Faraday rotator allow to tailor its working wavelength to arbitrary spectral positions within the transparency window of the magneto-optical slab. The demonstrated concept might lead to important, highly integrated, nonreciprocal, photonic devices for light modulation, optical isolation, and optical magnetic field sensing. Moreover, the simple fabrication of EuSe nanostructures by physical vapor deposition opens the way for many interesting magneto-plasmonic systems and 3D magneto-optical metamaterials. This section has been adapted with permission from the dissertation [43] and the article [40].

##### 4.1. The waveguide-plasmon-polaritonic approach

From the discussion of the Faraday effect in section 2.2.3, we have seen that, in first approximation, the Faraday rotation angle scales linearly with the applied magnetic field, the material specific Verdet constant and the material thickness. Hence, as the rotation capabilities of a magneto-optic material is usually limited by a small Verdet constant, a thin-film rotator structure might seem contradictory. However, a strong Faraday effect and a low material thickness can be combined by making the rotator structure resonant. As it was also discussed in section 2.2.3, the Faraday effect is non-reciprocal, which allows light to accumulate rotation of the same sign and magnitude for both forward and backward propagation. This behavior implies that when light propagates through a medium and is reflected in the backward direction, the accumulated Faraday rotation is twice the rotation of only one pass. Moreover, the rotation can be enhanced even further by additional round-trips through the medium. This principle is utilized in the magneto-plasmonic structure schematically depicted in figure 23(a). The structure consists of an EuSe slab waveguide and a gold wire grating on top. Both the magnetic field and the wave vector of the incident light are assumed to be perpendicular to the EuSe film. In the following, the incident polarization parallel and perpendicular to the wires is referred to as TE and TM, respectively. The gold nanowire grating has two functions. First, it acts as a waveguide coupler and allows normally incident light to couple into the slab and excite a waveguide mode (see section 2.6). The wave vectors of these modes possess a significant component in the  $z$ -direction. In ray approximation, this scenario can be understood as light bouncing back and forth inside the magneto-optical film and accumulating the rotations of multiple round-trips. When light is eventually coupled out of the film, due to a finite lifetime, the effective Faraday rotation is significantly larger than that for a bare film.

Many applications require wavelength-specific device operation. The presented structure design provides the freedom to tune the working wavelength (i.e. the spectral region with largest rotation enhancement) to arbitrary positions



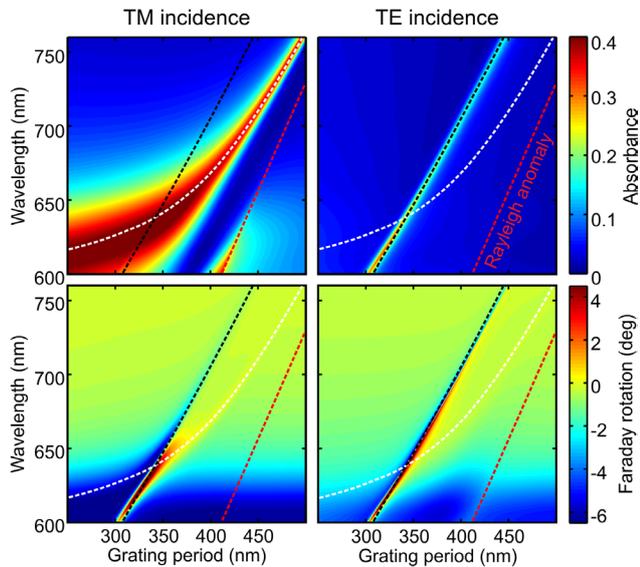
**Figure 23.** (a) Geometry of the EuSe-Au hybrid structure. The applied magnetic field and incident light are normal to the EuSe film. There are two cardinal polarization orientations, namely transverse electric (TE, electric field parallel to the gold wires) and transverse magnetic (TM, electric field perpendicular to the gold wires). (b) Schematic dispersion graph of the hybrid structure. For TM incident polarization, the plasmonic resonance of the gold wires couples strongly to the TM waveguide mode of the EuSe slab and forms a waveguide-plasmon-polariton. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

within the transparency window of the magneto-optical film. To explain the principle behind the wavelength tunability, we must first consider the connection between Faraday rotation enhancement and the dispersion properties of the hybrid structure. The largest Faraday rotation enhancement occurs at wavelengths at which the structure supports both a TM-polarized and a TE-polarized eigenmode (see section 2.6). In that case, the TM-to-TE conversion (i.e. polarization rotation) can occur most efficiently. This behavior will also be described analytically in section 5. Strictly speaking, the modes excited by TE- and TM-polarized incident light are themselves not purely (but mainly) TE- and TM-polarized due to the anisotropic permittivity tensor of the magneto-optical material. However, in the interest of readability, these modes are referred to as TE and TM modes, respectively. To overlap the TE and TM modes in  $k$ -space as well as at their energetic position, the second function of the metallic grating comes into play. In TM polarization, the gold wires provide a localized particle plasmon resonance that hybridizes with the TM waveguide mode of the magneto-optical slab and forms a waveguide-plasmon-polariton (WPP) [38]. For the resulting TM-polarized WPP, there will always exist a grating period

such that its dispersion curve intersects with a TE waveguide dispersion curve at the same wavelength. This mechanism is illustrated in figure 23(b), which schematically displays the dispersion behavior of the magneto-plasmonic hybrid structure. The lines in the diagram trace the resonance frequencies of the TE and TM modes for different grating periods. The solid lines correspond to the case when the grating coupler is made of a dielectric material and thus does not support any plasmonic resonance. As it was discussed in section 2.6, in the empty-lattice approximation, the waveguide resonance wavelength increases with larger grating periods. Since the exciting light wave is assumed to be impinging normally, the solid lines in the dispersion diagram can be associated with the energy  $E_1$  in figure 16. The dispersion for a metallic waveguide coupler is depicted as dotted lines. In the dielectric case, the TE and TM modes never overlap except for the zero grating period or inclined incidence [37], whereas they overlap in the metallic case owing to the formation of the WPP. As a result of the coupling of the localized particle plasmon to the TM waveguide mode, the dispersion curve of the TM mode is bent over the TE waveguide-mode dispersion curve and an intersection is created [37, 38]. The position of this intersection directly depends on the wavelength of the localized particle plasmon in the grating wires. By increasing (decreasing) the wire width of the grating, both the plasmonic resonance and intersection point of the WPP and TE mode shift toward longer (shorter) wavelengths and larger (smaller) periods. By means of this feature, the working wavelength of the system can be tuned via the gold wire width and grating period.

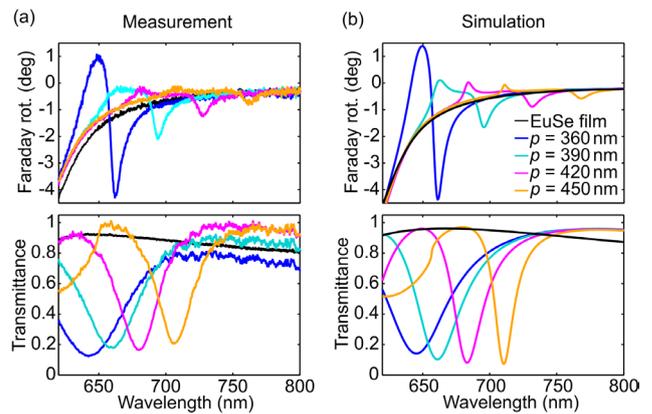
In the experimental realization of the hybrid-plasmonic structure, the waveguide was made of EuSe. Already in the 1960s, in the context of research on computer memories, the group of Europium chalcogenides received attention for their exceptional magnetic properties and strong magneto-optical response at low temperatures [59, 183, 184]. In particular the compound EuSe exhibits extremely large Faraday rotation angles on the order of  $1^\circ$  per Tesla and micrometer thickness in the visible wavelength range at temperatures of 30 K. EuSe also possesses a high saturation magnetic flux density of 2–5 T, depending on the wavelength [183–185]. Furthermore, the fabrication of EuSe thin films is very simple and can be carried out by physical vapor deposition [186] (PVD). This feature is a significant advantage over BIG, which is widely utilized both in magneto-optical devices [187–190] and concept studies [14, 36, 162]. BIG films are typically fabricated by pulsed laser deposition [60] (PLD) followed by high-temperature annealing, which is largely restricted to homogeneous films and does not allow for the direct incorporation of other materials, such as plasmonic nanostructures, into the film. With the flexibility of PVD, EuSe provides the possibility of fabricating more sophisticated potential future designs, including 3D geometries, where the magneto-optical and plasmonic elements are merged. The gold gratings attached to the EuSe films were fabricated by electron beam lithography [191].

To support the explanation of the Faraday rotation enhancement mechanism, the dispersion behavior of the demonstrated structures was simulated using the scattering matrix method [192]. The simulation results displayed in figure 24 confirm



**Figure 24.** Simulation of absorbance and Faraday rotation for a 150 nm thick EuSe slab and 70 nm thick and 70 nm wide periodic gold wires. The assumed conditions are a temperature of 30 K and a magnetic field of 5 T. The black and white dashed lines denote the calculated TE and TM resonance modes. At the point where TM and TE modes overlap, the largest Faraday rotation occurs. The red dashed line indicates the Rayleigh anomaly depending on the grating period. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

the formation of WPPs in the EuSe-Au hybrid structure for TM incidence and also show that the maximum Faraday rotation enhancement can be expected around the intersection of the TE waveguide mode and the TM-polarized WPP. The upper panels of figure 24 show the simulated absorbance spectra for a 150 nm thick EuSe film with a gold wire grating on top for varying the grating periods. The gold wires are assumed to be 70 nm thick and 70 nm wide. The black and white dashed lines trace the radiating TE- and TM-mode resonance frequencies of the EuSe hybrid structure and have been derived *ab initio* from the scattering matrix [192]. The diffraction induced Rayleigh anomaly [38] is plotted in red. For small periods, the TM hybrid-mode dispersion is dominated by the plasmonic resonance of the gold wires and is therefore relatively broad with a weak dependence on the grating period. For larger periods, the TM hybrid-mode dispersion becomes more waveguide-like and converges toward a sharp and purely waveguide-induced resonance. Because the localized particle plasmons in the gold wires can be excited only for TM-incident polarization, the TE-mode dispersion shows a distinct feature of a waveguide resonance. The corresponding Faraday rotation spectra are shown in the lower panels of figure 24. The maximum Faraday rotation occurs around the intersection of the TE and TM mode dispersion, as predicted in the previous discussion. For the case when the wave vector of the incident light is tilted, the dispersion behavior changes according to the analysis by Christ *et al* for non-magneto-optical hybrid structures [37, 38]. However, the criterion for maximum Faraday rotation enhancement remains unchanged: it occurs where the TM waveguide-plasmon-polariton has the strongest overlap with the TE waveguide modes. More discussion on that matter



**Figure 25.** Faraday rotation and transmittance spectra for a 150 nm thick EuSe slab and 70 nm thick and 70 nm wide periodic gold wires. By varying the grating period, the overlap of the TM and TE waveguide modes can be successively increased and the Faraday rotation enhanced. (a) Measured data. (b) Simulated data. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

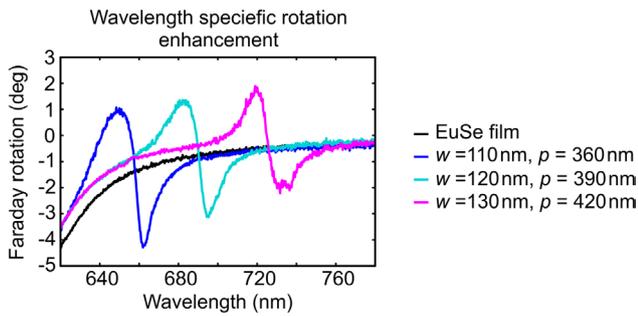
can be found in section 4.3, which will also elaborate on the influence of the waveguide thickness.

In the numerical simulations, the gold has been described by tabulated data by Johnson and Christy [193]. The glass substrate was assumed to possess a constant relative permittivity of 2.13. The diagonal elements of the permittivity tensor of the EuSe film were modeled using experimental data gained from transmission measurements of different plasmonic gratings on top of an EuSe slab. The off-diagonal elements of the dielectric tensor were obtained from measurements of a blank EuSe film at 30 K and 5 T, which is the condition given in the performed experiments.

#### 4.2. Experimental realization

The Faraday rotation measurements for the EuSe-Au hybrid structures were performed in a magnet cryostat at 30 K. The polarization rotation was measured with a rotating analyzer between the illuminated sample and a spectrometer. Because the presented structures are anisotropic, for rotation measurements it is crucial that the incident light be either purely TE- or TM-polarized. For other incident polarizations, undesired reciprocal rotation contributions occur owing to different transmittance for TE and TM polarization. Wavelength-dependent deviations from the pure TE or TM polarizations can occur due to the Faraday rotation in glass elements within the reach of the magnetic field of the magnet cryostat. For this reason, an intra-cryostat polarizer was placed directly in front of the sample to ensure that the unpolarized measurement light beam is only polarized just before it hits the sample.

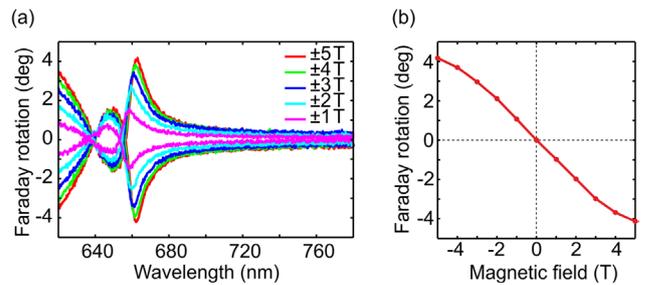
**4.2.1. Faraday rotation enhancement.** The simulated Faraday rotation of the EuSe-Au structures, presented in section 4.1, was subsequently confirmed experimentally. Figure 25(a) displays the transmittance and Faraday rotation spectra for TM incidence for different grating periods. The utilized EuSe film is 150 nm thick. Both the width and thickness of the gold wires is 70 nm. The corresponding



**Figure 26.** Experimental demonstration of sweeping the Faraday rotation enhancement over a range of wavelengths. The graph is showing the Faraday rotation of a 150 nm thick EuSe slab and 70 nm thick periodic gold wires. From left to right, the curves correspond to a wire width  $w$  of (70 nm, 80 nm, 90 nm) and a grating period  $p$  of (360 nm, 390 nm, 420 nm). Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

spectra from the simulations are plotted in figure 25(b). The black curves represent the case of a blank EuSe film without grating. The blue curves correspond to a 360 nm grating period and represent the case where the TE- and TM-mode dispersions have the greatest overlap. Hence, the largest Faraday rotation enhancement is achieved. At 662 nm, the structure exhibits a Faraday rotation of  $4.2^\circ$  at reasonably high transmittance of 30%. The other curves show that when the period is increased, the enhancement of the rotation decreases as the TE and TM modes move farther apart. As in the color-coded absorbance diagram in figure 24, the resonance features of the TE mode (negative rotation contribution) and TM mode (positive rotation contribution) are drifting away from each other with an increasing grating period while simultaneously flattening out. The simulated and experimental data correspond well, although there is a difference in the Faraday rotation base line plotted in black. For small wavelengths, in the simulation, this curve completely overlaps with the colored rotation spectra of the different gratings. However, in the measurement this is not completely the case. This discrepancy is caused by the limited temperature accuracy of the utilized cryostat system. In principle, this effect can be completely eliminated by using a cryostat that can maintain a temperature below 7 K, which is the Néel temperature of EuSe. Earlier measurements (not shown here) confirmed that in this temperature range, the Faraday rotation of EuSe is even larger and not temperature dependent.

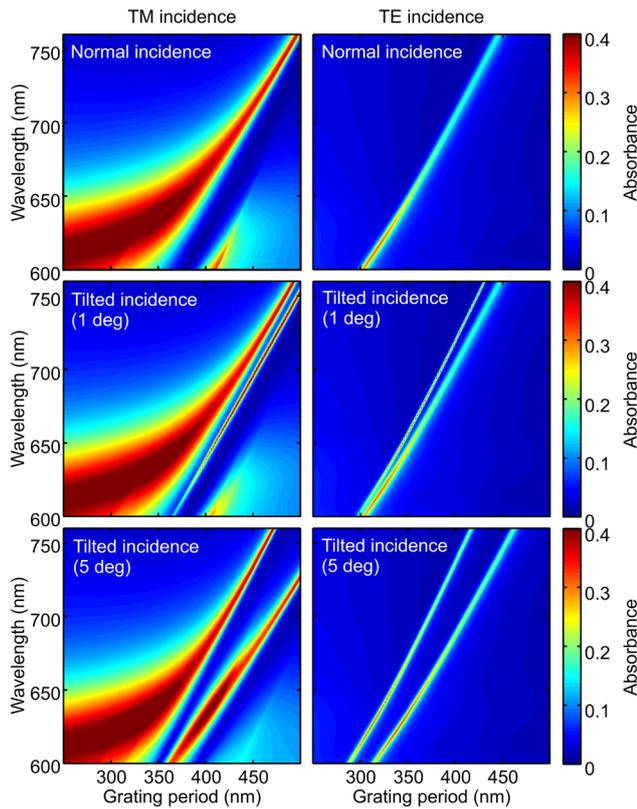
**4.2.2. Tunable working wavelength.** As explained in section 4.1, the Faraday rotation of the waveguide material can be enhanced at selected wavelengths. The only requirement is that the material offers a sufficiently low absorption at the wavelength of interest to achieve a high-quality factor of the waveguide resonator. From the visible to the near infrared, EuSe is transparent at wavelengths larger than 550 nm [194]. Figure 26 shows the measured Faraday rotation spectrum of a 150 nm thick EuSe film at a temperature of 30 K and a magnetic flux density of 5 T. The black curve denotes the Faraday rotation for the case without plasmonic enhancement. By tailoring the plasmonic gratings such that the TE



**Figure 27.** Magnetic tuning of the polarization rotation. (a) Faraday rotation of a 150 nm thick EuSe slab and 70 nm wide periodic gold wires for different applied magnetic fields. (b) Magnetic field dependence of the Faraday rotation at 663 nm. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

waveguide mode and TM WPP mode dispersions overlap at different wavelengths, the Faraday rotation can be enhanced selectively at these tailored wavelengths. As it was explained in section 4.1, for a given thickness of the magneto-optical slab, the TE-TM overlap can be achieved by tuning the wire width and period of the metal wires. The colored curves in figure 26 correspond to three different plasmonic gratings, which fulfill the TE-TM-matching condition. All gratings share a constant wire thickness of 70 nm. Both the wire width  $w = (70 \text{ nm}, 80 \text{ nm}, 90 \text{ nm})$  and the grating period  $p = (360 \text{ nm}, 390 \text{ nm}, 420 \text{ nm})$  are increasing from left to right. The resulting maximum absolute values of Faraday rotation occur at the wavelengths  $\lambda = (662 \text{ nm}, 695 \text{ nm}, 736 \text{ nm})$ . For longer wavelengths, both the intrinsic Faraday rotation of the film and the enhanced rotation are decreasing. This finding is in agreement with the assumption that the enhanced Faraday rotation and the intrinsic Faraday rotation of a film are approximately proportional to each other for constant absorption, a constant waveguide coupling efficiency, and thus a constant quality factor.

**4.2.3. Magnetically tunable polarization rotation.** The magnetic field dependence of the Faraday rotation was measured to demonstrate the polarization tuning capability of the demonstrated EuSe-Au structures. Figure 27(a) shows the Faraday rotation of the structure with a 360 nm grating period, 70 nm wire thickness, and 70 nm wire width for TM incidence. The magnetic field was varied from -5 T to +5 T. The structure geometry yields largest Faraday rotation at 663 nm. For this wavelength, a polarization rotation tuning range from  $-4.2^\circ$  to  $+4.2^\circ$  is obtained. The tuning behavior is nearly linear in the magnetic field, as can be extracted from figure 27(b). The polarization rotation measurement for inverted magnetic fields also acts as a control experiment to reveal potential spurious non-magnetically induced contributions. Since the measured samples are anisotropic, such effects can occur when the incident polarization deviates from an exactly TE or TM polarized state, for instance, as a result of the Faraday rotation of the cryostat windows. The highly mirror symmetric behavior of the measured rotation spectra in figure 27(a) clearly shows that polarization errors of the incident light are well under control.

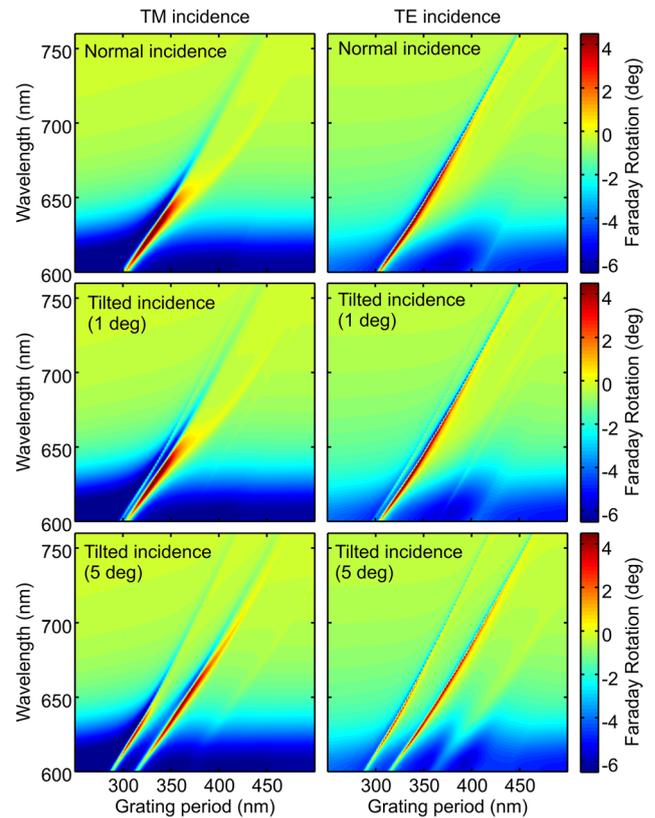


**Figure 28.** Dispersion of the absorbance for different angles of incidence. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

#### 4.3. On the incident angle and film thickness

The simulations and measurements presented in the sections 4.1 and 4.2 were performed for normal incidence. Here, the case of tilted incidence will be discussed by means of numerical simulations. As in the previous sections, the hybrid structure is assumed to consist of a 150 nm thick EuSe slab with 70 nm thick and wide gold wires on top. Furthermore, also a magnetic field of 5 T and a temperature of 30 K are assumed.

The optical path length through a magneto-optical film can be increased by tilting the incident beam. However, the Faraday rotation is proportional to the component of the optical path in the direction of the magnetization of the material, which is perpendicular to the EuSe film. Therefore, a small tilting angle does not affect the polarization rotation in a bare film. However, a different behavior is exhibited by the presented magnetoplasmonic hybrid structures. Their WPP dispersion, and thus, also the Faraday rotation response critically depends on the incident angle. This is illustrated by the absorbance plots in figure 28 for both TM and TE polarized incident light, as well as for different angles of the wavevector in the plane perpendicular to the gold wires. It can be clearly seen that for an increasing tilting angle, there emerges a second resonance feature both for TM and TE incidence. This is in agreement with the analysis of non-magneto-optic WPP systems by Christ *et al* [37, 38], showing that for normal incidence there are two TE and TM polarized waveguide modes with similar frequency, where one of them is symmetry-forbidden

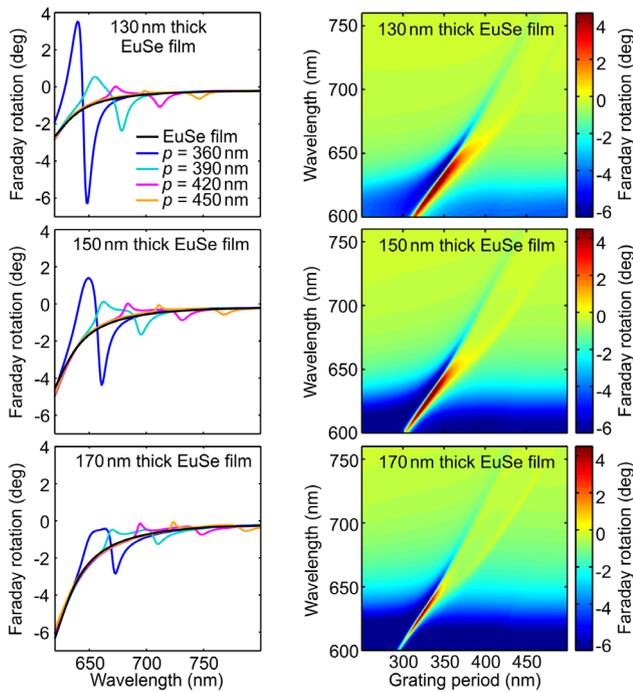


**Figure 29.** Dispersion of the Faraday rotation for different angles of incidence. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

and thus possesses zero line-width. However, by deviating from normal incidence the dark waveguide modes become bright. Furthermore, with an increasing tilting angle, the two resonances move further away from each other. In the empty lattice picture, illustrated in figure 16 for the TE modes, this behavior can be understood in the following way: for normal incidence ( $k_x = 0$ ) the two branches of the TE mode overlap at energy  $E_1$ . This degeneracy is lifted when the tilting angle is increased ( $k_x > 0$ ).

Comparing the modal dispersion in figure 28 with the corresponding Faraday rotation plots in figure 29 reveals that, regardless of the more complex spectra for tilted incidence, the criterion for maximum Faraday rotation enhancement is still the same. It occurs where the TM WPP has the strongest overlap with the TE waveguide modes.

It is also interesting to investigate the influence of the thickness of the magneto-optical EuSe slab. Since the Faraday rotation of a bulk piece of material scales with its thickness, it might seem reasonable that a thicker EuSe film also leads to larger Faraday rotation. However, this is not generally the case, especially when the grating period and the wire dimensions are kept the same. To illustrate this, let us consider the dispersion diagram in figure 23(b): increasing the film thickness induces a redshift of the waveguide modes, whereas the plasmonic resonance remains at the same spectral position (in first approximation). As a result, the TE-TM overlap region shifts to smaller periods. The consequence of this behavior is illustrated in figure 30, which shows the simulated Faraday



**Figure 30.** Dispersion of the Faraday rotation for different EuSe film thicknesses. The incident light is TM polarized. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

rotation dispersion spectra for different film thicknesses and different periods. The gold wires are assumed to be 70 nm thick and wide. The simulations confirm that the Faraday rotation of the film alone (far away from the TE and TM modes) increases for thicker EuSe films. However, since the TE-TM-overlap region is slightly shifted toward smaller periods, the Faraday rotation decreases for the listed periods. If desired, in principle, the shift of the TE-TM overlap could be compensated by tuning the plasmonic resonance toward longer wavelengths, which can be achieved by increasing the wire width.

#### 4.4. Conclusion

An actively tunable thin-film optical rotator with a variable working wavelength was demonstrated both in experiment and in simulation. This was achieved by combining a slab of EuSe, which possesses a large Verdet constant and high saturation magnetic flux density, with a gold nanowire grating. The wavelength range of largest Faraday rotation can be selected by choosing the correct combination of grating period and wire width. The magnitude of optical rotation can be magnetically tuned over a wide angular range. At 30 K for a 220 nm thick structure, a rotation tuning range of up to  $8.4^\circ$  was obtained. For the present structure, this range can presumably be doubled when cooling down to temperatures below the Néel temperature of EuSe, which is 7 K.

The demonstrated concept can be expected to have applications in highly integrated optics, demanding actively controlled optical modulation [195], magnetic field sensing [196, 197] and optical isolation [12–14, 16–18]. Furthermore, the presented structure geometry is suitable for large-area

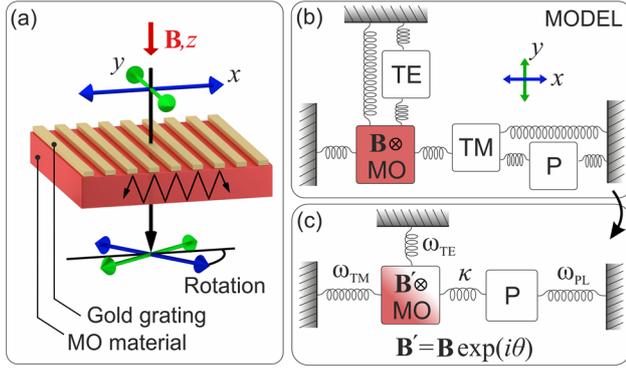
fabrication [198, 199], which makes it a promising design for non-reciprocal coatings of optical elements, such as lenses, with active external control at specific wavelengths. In addition, the direct attachment onto optical fiber ends [200–202] or onto laser diodes could yield devices with extremely small volumes for a highly integrated environment.

The working principle of the presented structures is not restricted to EuSe as a waveguide material. It can be directly transferred to other magneto-optical materials. The only requirement is that the magneto-optical material provides a transparency window near the wavelength region of interest and a sufficiently high surface quality to ensure a high  $Q$ -factor. These requirements can be met by commonly used room temperature magneto-optical materials, such as BIG [60], YIG [60] or TGG [203]. For the low-temperature regime, there are also other chalcogenides, such as EuS [183], EuTe [184], EuO [204], which have similar magneto-optical properties as EuSe. The temperature at which these materials show the largest Faraday rotation can possibly be increased by doping with Gd. For example, doping EuO with Gd can raise the Curie temperature from 69 K to 135 K [57, 205]. EuSe and EuS combine large magneto-optical response, high saturation magnetic flux density, and simple thin-film fabrication via PVD. Thus, they are promising materials for further magnetoplasmonic studies with structures that are not restricted to designs including strictly continuous magneto-optical films. For example, in section 6, EuS will be utilized for a less than 200 nm thick structure design, that relies on an EuS thin film with an incorporated gold grating and produces Faraday rotation of up to  $14^\circ$ , corresponding to a tuning range of over  $25^\circ$ .

## 5. Lorentz force model for magnetoplasmonics

It was shown recently that the Faraday rotation [36] and also the transverse magneto-optic Kerr effect [34] of a dielectric film can be enhanced considerably by attaching a resonant plasmonic grating. As it is demonstrated in section 4, by varying the grating and nanowire geometry, the maximal polarization rotation enhancement can be tuned to arbitrary spectral positions [40]. Such structures exhibit Faraday rotation of up to  $4.2^\circ$  for a thickness of 220 nm [40]. Hence, they are very relevant for future devices, such as thin-film Faraday rotators and isolators as their performance exceeds other approaches considerably.

While the experimental realization and numerical simulation of such systems received considerable attention, so far, there has not been an analytical theoretical description. In this section, a simple coupled oscillator model will be presented, that reveals the underlying physics inside hybrid magnetoplasmonic systems and yields analytical expressions for the resonantly enhanced magneto-optical response. The Lorentz nonreciprocity of the oscillator model is intrinsically incorporated via the Lorentz force, which is proportional to  $\mathbf{v} \times \mathbf{B}$ . Moreover, the predictions of the model are in good agreement with rigorous numerical solutions of Maxwell's equations for typical sample geometries. The demonstrated ansatz is transferable to other complex and hybrid nano-optical systems and will significantly facilitate device design.



**Figure 31.** (a) Schematic drawing of the hybrid magnetoplasmonic nanostructure. (b) Mechanical analog that represents the coupling of the relevant optical excitations. (c) Simplified oscillator model providing analytical solutions. Adapted figure with permission from [41], Copyright (2016) by the American Physical Society.

In section 5.1 the model will be introduced and applied to the EuSe-Au structures discussed in the previous section. Next, the model will be assessed by comparing its predictions to full numerical simulations based on Maxwell's equations. In order to maintain good readability, some of the mathematical details of the model are presented separately in the section 5.2.

This section has been adapted with permission from the dissertation [43] and from the article [41], Copyright (2016) by the American Physical Society.

### 5.1. Application to EuSe-Au hybrid structures

The geometry of the investigated magneto-optical system is depicted in figure 31(a). It consists of a dielectric magneto-optic (MO) thin film with an attached one-dimensional gold grating on top. The incident light is assumed to be linearly polarized and impinges on the sample along the  $z$ -direction. The polarization direction with electric field perpendicular (parallel) to the gold wires will be referred to as  $x$ - ( $y$ -) polarized. Figure 31(b) depicts the corresponding mechanical analog of the optical system, where each relevant optical excitation is represented by a mass suspended by a linear spring. The coupling between the different excitations is modeled by interconnecting springs. All masses are assumed to be charged and driven by the external light field. Due to its periodicity, the gold grating acts primarily as a waveguide coupler and allows for the far-field excitation of transverse electric (TE) and transverse magnetic (TM) polarized quasi-guided waveguide modes inside the MO film. In the absence of the magnetic field, the TE (TM) waveguide modes can only be excited by  $y$ - ( $x$ -) polarized incident light. Thus, the waveguide modes are modeled by one mass each, which is restricted to move only along the  $x$ - or  $y$ -direction.

The second purpose of the gold wires is to provide a localized plasmon resonance, which can be excited by  $x$ -polarized light. This plasmonic resonance is taken into account by an oscillator moving in  $x$ -direction (labeled P). Due to the field overlap, the plasmonic resonance is coupled to the TM

waveguide mode [37, 38]. The dielectric response of the MO material itself is modeled by the red mass, which can move within the  $xy$ -plane and is subjected to a Lorentz force [50] in the  $xy$ -plane due to a static magnetic field  $\mathbf{B}$  oriented along  $z$ -direction.

The oscillator system in figure 31(b) possesses five degrees of freedom. Its motion is described by five coupled second order linear differential equations. While these equations can be solved exactly, it is impossible to derive closed expressions for its eigenmode frequencies. To simplify the model and allow for the analytical calculation of the eigenmode frequencies, a series of proper approximations can be applied.

First of all, neglecting the dispersion effects by the MO material itself yields significant simplification. This is achieved by assuming the driving frequency to be far away from the MO oscillator resonance and the impact of the driving force on the MO oscillator (i.e. the associated coupling constant) to be small, resulting in a reduced system of three coupled second order equations. Furthermore, in a rotating wave approximation, which is valid when the driving frequency  $\omega$  is close to the eigenfrequencies  $\Omega_j$ , ( $j = \text{TE}, \text{TM}, \text{P}$ ) of the individual oscillators, the second order equations are reduced to first order. The mathematical details of the model reduction are provided in section 5.2 and the limitations of the applied approximations are discussed later in this section.

The simplified oscillator scheme is depicted in figure 31(c). The three masses of the TE, TM and material oscillator are now merged into one waveguide oscillator. Assuming a time-harmonic oscillator displacement that is proportional to  $\exp(-i\omega t)$ , the governing equations in the rotating wave approximation are given by the matrix equation

$$(M_0 + \Delta M - I\omega)\mathbf{x} = \eta R \mathbf{E}_{||}, \quad (104)$$

where  $I$  is the  $3 \times 3$  identity matrix, and  $\eta$  is a residue of the rotating wave approximation that is inversely proportional to the effective mass and the density of the oscillators (see section 5.2). The vector  $\mathbf{x} = (x_{\text{TM}}, x_{\text{P}}, y_{\text{TE}})^T$  contains the displacements of the corresponding oscillators in the  $xy$ -plane, while  $\mathbf{E}_{||} = (E_x, E_y)^T$  denotes the driving electric field. The forces acting on each oscillator are proportional to  $R \mathbf{E}_{||}$ , with the charge density matrix

$$R = \begin{pmatrix} \rho_{\text{TM}} & 0 \\ \rho_{\text{P}} & 0 \\ 0 & \rho_{\text{TE}} \end{pmatrix}. \quad (105)$$

Furthermore,  $M_0$  accounts for the coupling of the TM waveguide mode and the plasmon [37], with

$$M_0 = \begin{pmatrix} \omega_{\text{TM}} & -\kappa & 0 \\ -\kappa & \omega_{\text{P}} & 0 \\ 0 & 0 & \omega_{\text{TE}} \end{pmatrix}. \quad (106)$$

The corresponding coupling constant  $\kappa$  is assumed to be purely real, while  $\omega_j = \Omega_j - i\Gamma_j$ , ( $j = \text{TE}, \text{TM}, \text{P}$ ) are complex frequencies that consist of the resonance frequencies  $\Omega_j$  and the damping coefficients  $\Gamma_j$  (due to radiative and absorptive losses) of the different modes. The antisymmetric matrix

$\Delta M$  denotes the nonreciprocal influence of the magnetic field via the Lorentz force proportional to  $\mathbf{v} \times \mathbf{B}$  and is defined as

$$\Delta M = \beta e^{i\theta} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix}, \quad (107)$$

with the real coupling coefficient  $\beta$ , that is proportional to the static magnetic field. The factor  $\exp i\theta$  stems from the four-oscillator model and represents the phase of the MO oscillator in that system. In the supplement of [41] it is shown by perturbation theory [206, 207] that this phase corresponds to the phase of the gyration  $g = |g| \exp(i\theta)$  of the MO material.

The optical response of the system is obtained by assigning an effective susceptibility to the system. This is done by summing up the effective electronic polarization  $\mathbf{P}_{||} = \chi \mathbf{E}_{||}$ , which can be identified as  $\mathbf{P}_{||} = R^T \mathbf{x}$ . Hence, the effective susceptibility can be written as

$$\chi(\omega) = \eta R^T M(\omega)^{-1} R, \quad (108)$$

with  $M(\omega) = M_0 + \Delta M - I\omega$ . Due to the cross product in the Lorentz force,  $\Delta M$  and thus  $M(\omega)$  become antisymmetric for non-zero magnetic fields, reflecting the nonreciprocity of the system [4, 48].

The eigenfrequencies of the coupled oscillator system are obtained by setting the external electric field in equation (104) to zero. This results in the following eigenvalue problem

$$(M_0 + \Delta M) \mathbf{x}_n = \omega_n \mathbf{x}_n, \quad (109)$$

where  $\omega_n$  denotes the eigenvalues, and  $\mathbf{x}_n$  the eigenvectors for  $n = 1, 2, 3$ . In the presented model, the Lorentz force is assumed to be weak compared to the restoring forces. Hence,  $\Delta M$  is regarded as a small perturbation of  $M_0$ , resulting in  $\omega_n$  being close to the eigenfrequencies of  $M_0$ . The eigenfrequencies of  $M_0$  are given by

$$\omega_{1/2} = \frac{\omega_{\text{TM}} + \omega_{\text{p}}}{2} \mp \sqrt{\kappa^2 + \left(\frac{\omega_{\text{TM}} - \omega_{\text{p}}}{2}\right)^2}, \quad (110)$$

$$\omega_3 = \omega_{\text{TE}}. \quad (111)$$

The first two eigenfrequencies correspond to the two branches of a WPP hybrid mode arising from the coupling between the plasmonic mode and the TM waveguide mode [37, 38]. The third eigenfrequency is simply the frequency of the TE polarized waveguide mode. In section 4, it was demonstrated numerically that the largest magneto-optical response occurs for grating periods at which the TE waveguide mode and one of the TM polarized WPP branches possess similar resonance frequencies, i.e. when  $\omega_{1/2} = \omega_{\text{TE}}$ . This behavior can now be deduced analytically from the presented model by examining the inverse of  $M$  for a small perturbation  $\Delta M$ :

$$M^{-1} \approx (M_0^{-1} - I\omega) - \frac{i\beta e^{i\theta}}{(\omega_1 - \omega)(\omega_2 - \omega)(\omega_{\text{TE}} - \omega)} \times \begin{pmatrix} 0 & 0 & +(\omega - \omega_{\text{p}}) \\ 0 & 0 & -\kappa \\ -(\omega - \omega_{\text{p}}) & +\kappa & 0 \end{pmatrix}. \quad (112)$$

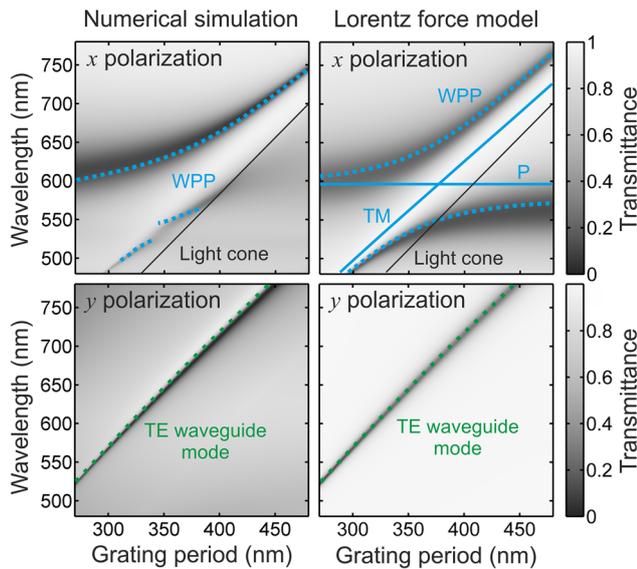
This expression reveals that the magnetic terms proportional to  $\beta$  become largest for  $\omega = \omega_{\text{TE}} = \omega_{1/2}$ , which confirms previous numerical findings.

To obtain the effective susceptibility for a particular nanostructure, the free parameters in  $M$  and  $R$  have to be deduced by a systematic and rigorous fitting procedure. The fitting process consists of three steps, in which  $M_0$ ,  $R$ , and  $\Delta M$  are fitted sequentially. Full numerical simulations based on the scattering matrix method [192, 208] were used as reference. In the following, the three fitting steps are discussed and applied to a sample geometry that consists of a 150 nm thick EuSe film with 70 nm thick and 70 nm wide gold wires on top (see figure 31(a)). The substrate under the film is assumed to be glass with a permittivity  $\varepsilon = 2.13$ . The applied magnetic field is assumed to be 5 T. The permittivity of the gold grating is modeled using the Drude model function  $\varepsilon(\omega) = \varepsilon_{\infty} - \omega_{\text{p}}^2 / (\omega^2 + i\gamma\omega)$ , which was introduced in section 2.5.1. The model parameters for gold were extracted from [63] and set to  $\varepsilon_{\infty} = 7.9$ ,  $\omega_{\text{p}} = 8.77$  eV, and  $\gamma = 1.13 \times 10^{14}$  s<sup>-1</sup>. The components of the permittivity tensor

$$\varepsilon_{\text{EuSe}} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ -\varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ -\varepsilon_{13} & -\varepsilon_{23} & \varepsilon_{33} \end{pmatrix} \quad (113)$$

of the magneto-optical slab are assumed to be  $\varepsilon_{11} = \varepsilon_{22} = 4.95 + 0.007i$ ,  $\varepsilon_{33} = 3.9643 + 0.0126i$ ,  $\varepsilon_{12} = 0.061 + 0.061i$ , and  $\varepsilon_{13} = \varepsilon_{23} = 0$ . These values correspond to a 150 nm thick EuSe thin film at a wavelength of 660 nm and a magnetic field of 5 T in  $z$ -direction. These data were obtained by previous magneto-optical measurements [40].

In the first step, the free parameters in  $M_0$  are fitted such that the eigenfrequencies of the mechanical system match the eigenfrequencies of the actual nanostructure obtained by rigorous numerical solutions of Maxwell's equations. While equation (111) allows the direct derivation of  $\omega_{\text{TE}}$ , the complex coefficients  $\omega_{\text{TM}}$ ,  $\omega_{\text{p}}$ , and the real coefficient  $\kappa$  in equation (110) cannot be deduced directly from the full numerical simulations. This can be resolved by calculating the real part of  $\omega_{\text{TM}}$  from an empty lattice approximation [72] and assuming  $\Gamma_{\text{TM}} \approx \Gamma_{\text{TE}}/10$ , as justified by the results by Christ and coworkers [37, 38]. Figure 32 shows the comparison between the simulated and fitted eigenmodes of the oscillator model that includes the Lorentz force. The eigenmodes are plotted as blue (TM) and green (TE) dashed lines in units of wavelength. The solid black lines mark the edge of the light cone (i.e. the Rayleigh anomaly). The blue solid lines correspond to the frequencies of the individual oscillators. Especially in the most relevant region around the intersection of the TE waveguide mode and WPP, the modal dispersion is reproduced very well by the oscillator model. Outside this region, in the simulated TM dispersion at around 530 nm, there is a discontinuity due to the presence of another spectrally close higher-order mode. As expected [209], the eigenmodes of the actual nanostructure exhibit a cut-off at the edge of the light cone (see also section 2.6). Since the model does not take the periodic geometry

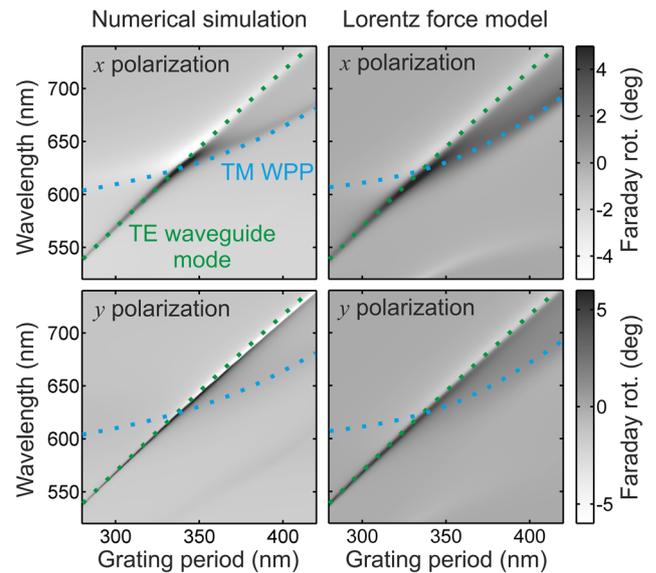


**Figure 32.** Comparison between transmittance spectra and modal dispersion derived from numerical simulations (left panels) and the Lorentz force oscillator model (right panels). TM (TE) polarized modes are plotted as blue (green) dashed lines. Blue solid lines indicate the uncoupled plasmon and TM waveguide frequencies. Adapted figure with permission from [41], Copyright (2016) by the American Physical Society.

into account, this discontinuity is missing in the dispersion plot of the model.

In the second part of the fitting sequence, the coefficients in  $R$  are derived by the condition that the transmittance of the effective medium has to reproduce that of the actual sample. In analogy to the first fitting step, it is assumed that the magnetic field only has weak influence on the absorption behavior. Hence, the absorption can be derived from the effective susceptibility for zero B-field. This is done by setting  $\Delta M = 0$  and solving the Helmholtz equation to obtain the evolution of an  $x$ - and  $y$ -polarized plane wave over an effective propagation distance (see section 2.2.2). The effective propagation distance was assumed to be the sample thickness of 220 nm. For the sake of simplicity, the coefficients in  $R$  were assumed to be constant for all grating periods. By comparing the simulated transmittance in figure 32 (left panels) and the modeled transmittance (right panels), it can be seen that, except for the discontinuities outside the region of interest, as discussed above, the line shapes agree very well.

In the last part of the fitting procedure, the remaining magneto-optical coefficients in  $\Delta M$  are determined. Far away from the MO material resonance, the gyration  $g = |g| \exp(i\theta)$  can be assumed to be constant. For bismuth iron garnet [36] and EuSe [40], this assumption is valid for the red and near-IR spectral region, where  $\theta \approx -45$  deg. This is also the value used for the oscillator model and the numerical simulations. Note that the model will work as well for other materials exhibiting different  $\theta$  (see supplemental material in [41]). The last remaining fitting parameter is  $\beta$ , which is proportional to the magnetic field. For realistic magnetic field strengths, its value only influences the magnitude of the MO response but not its spectral line shape. Thus,  $\beta$  is scaled such that the MO



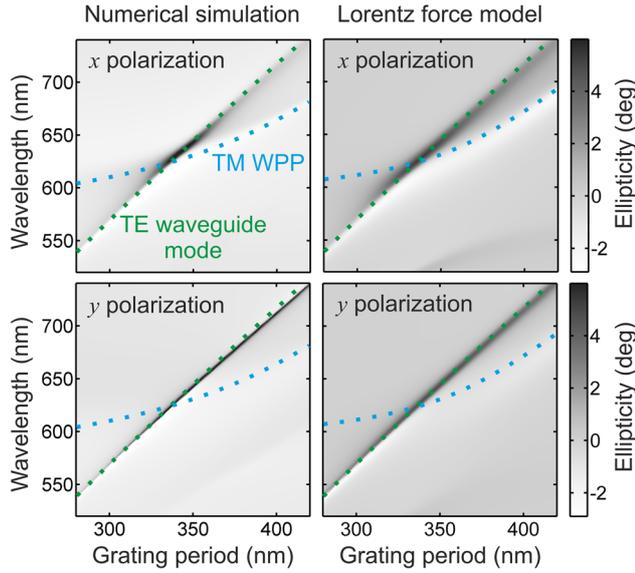
**Figure 33.** Comparison of the Faraday rotation derived from numerical simulations (left panels) and the Lorentz force oscillator model (right panels). The eigenfrequencies corresponding to the upper TM polarized waveguide plasmon polariton (WPP) branch and the TE waveguide mode are drawn as dashed lines. Adapted figure with permission from [41], Copyright (2016) by the American Physical Society.

response of the modeled system reaches the values of the full simulation.

Figure 33 depicts the comparison of the resulting Faraday rotation spectra from the full numerical simulations and the oscillator model. The green and blue dotted lines trace the eigenmodes. As expected by examining equation (112), the MO response is largest around the intersection point of the TM polarized WPP and the TE waveguide mode. Furthermore, the qualitative and quantitative agreement between model and numerical simulation is excellent.

The corresponding dispersion of the ellipticity response is displayed in figure 34. It can be seen, that the ellipticity predicted by the model is also in good agreement with numerical simulation. As in the case of the Faraday rotation dispersion, it can be seen that the ellipticity response is strongest around the intersection region of the TE waveguide mode and the TM polarized waveguide-plasmon-polariton mode, and lowest far-off resonance.

To compare the exact line shapes of the MO spectra, figure 35 displays the corresponding slice cuts of the Faraday rotation and ellipticity spectra. The model reproduces the line shapes very well. This includes the characteristic up-down feature in the Faraday rotation for  $x$ -polarization (indicated by arrows), which is successively spreading for periods larger than the period of the TE-WPP intersection (green line) as it was also described in section 4. On the other hand, the modeled MO spectra do not reproduce the offset in the numerically simulated spectra. This is a result of the approximations required for the reduction of the four-oscillators-model to the two-oscillators model. There, the oscillator strength of the MO material was assumed to be much smaller than the oscillator strength of the waveguide modes and the plasmon. As a result,



**Figure 34.** Comparison of the ellipticity derived from numerical simulations (left panels) and the Lorentz force oscillator model (right panels). The eigenfrequencies corresponding to the upper TM polarized waveguide plasmon polariton (WPP) branch and the TE waveguide mode are drawn as dashed lines. Adapted figure with permission from [41], Copyright (2016) by the American Physical Society.

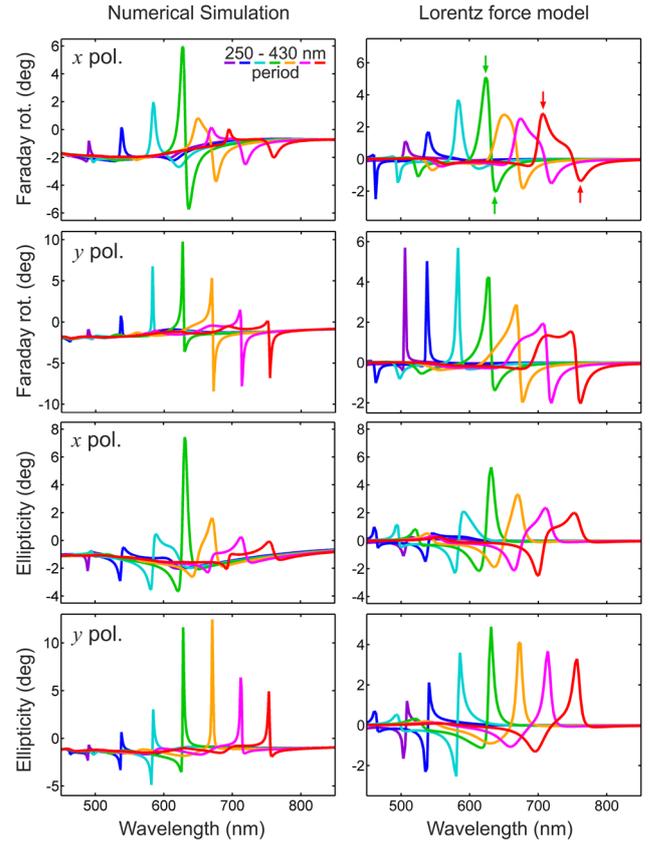
only the resonant contributions to the MO response are taken into account. The slightly broader MO features result from fitting the resonance linewidths to the values from numerical simulations. A better qualitative agreement in Faraday rotation could be obtained when adjusting the losses in the model independently of the calculated resonance line widths. In summary, it is evident that even the simplified oscillator model correctly predicts the overall shape of the MO spectra very well.

## 5.2. Simplifications and approximations

In this section, the equation of motion (104) of the simplified oscillator model will be derived explicitly. The discussion will start from the equations of motions of the extended oscillator model with five degrees of freedom (figure 31(b)) described by five coupled second order linear differential equations. By applying appropriate simplifications the system of five equations can be reduced to three coupled quadratic equations, which describe the motion of effectively three oscillators. Furthermore, in order to obtain an analytical solution of the three-oscillator problem, a rotating wave approximation will be applied.

**5.2.1. Extended model (5 degrees of freedom).** For light propagating in  $z$  direction ( $E_z = 0$ ), the equations of motion for the four-oscillators model in figure 31(b) are given by

$$X_{\text{MO}} - k_{\text{MO, TM}} x_{\text{TM}} - k_m \dot{y}_{\text{MO}} = \frac{q_{\text{MO}}}{m} E_x e^{-i\omega t} \quad (114a)$$



**Figure 35.** Comparison of the Faraday rotation derived from numerical simulations (left panels) and the Lorentz force oscillator model (right panels). The spectra correspond to slice cuts from figure 33 at equidistant periods from 250 nm to 430 nm with 30 nm spacing. Adapted figure with permission from [41], Copyright (2016) by the American Physical Society.

$$Y_{\text{MO}} - k_{\text{MO, TE}} y_{\text{TE}} + k_m \dot{x}_{\text{MO}} = \frac{q_{\text{MO}}}{m} E_y e^{-i\omega t} \quad (114b)$$

$$X_{\text{TM}} - k_{\text{MO, TM}} x_{\text{MO}} - k_{\text{P, TM}} x_{\text{P}} = \frac{q_{\text{TM}}}{m} E_x e^{-i\omega t} \quad (114c)$$

$$Y_{\text{TE}} - k_{\text{MO, TE}} y_{\text{MO}} = \frac{q_{\text{TE}}}{m} E_x e^{-i\omega t} \quad (114d)$$

$$X_{\text{P}} - k_{\text{P, TM}} x_{\text{TM}} = \frac{q_{\text{P}}}{m} E_x e^{-i\omega t}, \quad (114e)$$

where the lower case variables  $x_j$  and  $y_j$  with ( $j = \text{MO}, \text{P}, \text{TE}, \text{TM}$ ) denote the displacements of the oscillators associated with the magneto-optical (MO) material, the plasmon mode (P), as well as the TE and TM waveguide modes in  $x$  and  $y$  direction. The upper case symbols  $X_j$  and  $Y_j$  denote terms of the form

$$X_j = \ddot{x}_j + \Omega_j^2 x_j + 2\Gamma_j \dot{x}_j \quad (115a)$$

$$Y_j = \ddot{y}_j + \Omega_j^2 y_j + 2\Gamma_j \dot{y}_j \quad (115b)$$

with ( $j = \text{MO}, \text{P}, \text{TE}, \text{TM}$ ). Dots indicate a time derivative. The coefficient  $k_m$  is proportional to the magnetic field. Due to the cross product in the Lorentz force  $\propto(\mathbf{v} \times \mathbf{B})$ , it occurs in the first two equations with opposite signs. In order to reduce the number of parameters in the model, the oscillator masses  $m$  are assumed to be equal for all oscillators. The time harmonic ansatz  $\exp(-i\omega t)$  for the displacements leads to

$$W_{\text{MO}}(\omega)x_{\text{MO}} - k_{\text{MO,TM}}\dot{x}_{\text{TM}} + i\omega k_m y_{\text{MO}} = \frac{q_{\text{MO}}}{m}E_x \quad (116a)$$

$$W_{\text{MO}}(\omega)y_{\text{MO}} - k_{\text{MO,TE}}\dot{y}_{\text{TE}} - i\omega k_m x_{\text{MO}} = \frac{q_{\text{MO}}}{m}E_y \quad (116b)$$

$$W_{\text{TM}}(\omega)x_{\text{TM}} - k_{\text{MO,TM}}x_{\text{MO}} - k_{\text{P,TM}}x_{\text{P}} = \frac{q_{\text{TM}}}{m}E_x \quad (116c)$$

$$W_{\text{TE}}(\omega)y_{\text{TE}} - k_{\text{MO,TE}}y_{\text{MO}} = \frac{q_{\text{TE}}}{m}E_y \quad (116d)$$

$$W_{\text{P}}(\omega)x_{\text{P}} - k_{\text{P,TM}}x_{\text{TM}} = \frac{q_{\text{P}}}{m}E_x, \quad (116e)$$

where  $W_j(\omega) = -\omega^2 + \Omega_j^2 - 2i\Gamma_j\omega$ , ( $j = \text{MO}, \text{TM}, \text{TE}, \text{P}$ ). In matrix form, equation (116e) reads

$$M(\omega) \mathbf{x} = \frac{1}{m} \mathbf{Q} \mathbf{E}_{\parallel}. \quad (117)$$

Here, we introduced the vectors  $\mathbf{x} = (x_{\text{MO}}, y_{\text{MO}}, x_{\text{TM}}, y_{\text{TE}}, x_{\text{P}})^T$  as well as  $\mathbf{E}_{\parallel} = (E_x, E_y)^T$  and the matrices

$$M(\omega) = \begin{bmatrix} W_{\text{MO}}(\omega) & +i\omega k_m & -k_{\text{MO,TM}} & 0 & 0 \\ -i\omega k_m & W_{\text{MO}}(\omega) & 0 & -k_{\text{MO,TE}} & 0 \\ -k_{\text{MO,TM}} & 0 & W_{\text{TM}}(\omega) & 0 & -k_{\text{P,TM}} \\ 0 & -k_{\text{MO,TE}} & 0 & W_{\text{TE}}(\omega) & 0 \\ 0 & 0 & -k_{\text{P,TM}} & 0 & W_{\text{P}}(\omega) \end{bmatrix} \quad (118)$$

and

$$\mathbf{Q} = \begin{pmatrix} q_{\text{MO}} & 0 \\ 0 & q_{\text{MO}} \\ q_{\text{TM}} & 0 \\ 0 & q_{\text{TE}} \\ q_{\text{P}} & 0 \end{pmatrix}. \quad (119)$$

The optical response of the system is obtained by assigning an effective susceptibility to the system. This is done by summing up the effective electronic polarization  $\mathbf{P}_{\parallel} = \chi \mathbf{E}_{\parallel}$ , which is given by  $\mathbf{P}_{\parallel} = n \mathbf{Q}^T \mathbf{x}$ , where  $n$  denotes the effective oscillator density. Hence, the effective susceptibility can be written as

$$\chi(\omega) = \frac{n}{m} \mathbf{Q}^T M(\omega)^{-1} \mathbf{Q}. \quad (120)$$

**5.2.2. Identifying the gyration of the waveguide.** The following auxiliary calculation will establish a relation between the gyration of the MO film and the model parameters. The special case of a bare magneto-optical slab is described by

equations (116a) and (116b) for  $k_{\text{MO,TM}} = k_{\text{MO,TE}} = 0$ . In analogy to equation (117), the corresponding matrix form is

$$\begin{bmatrix} W_{\text{MO}}(\omega) & +i\omega k_m \\ -i\omega k_m & W_{\text{MO}}(\omega) \end{bmatrix} \begin{pmatrix} x_{\text{MO}} \\ y_{\text{MO}} \end{pmatrix} = \frac{q_{\text{MO}}}{m} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \quad (121)$$

For small  $k_m$ , i.e. for small magnetic fields, the resulting susceptibility is then given by

$$\chi(\omega) = \frac{n q_{\text{MO}}^2}{m W_{\text{MO}}(\omega)} \begin{pmatrix} 1 & \frac{-ik_m\omega}{W_{\text{MO}}(\omega)} \\ \frac{+ik_m\omega}{W_{\text{MO}}(\omega)} & 1 \end{pmatrix}. \quad (122)$$

As discussed in section 2.2.2, for light propagating in  $z$  direction, the susceptibility tensor  $\chi = \varepsilon - I$  of a MO material can be written as

$$\chi = \begin{pmatrix} \chi_{xx} & +ig \\ -ig & \chi_{yy} \end{pmatrix}, \quad (123)$$

where  $g$  is the complex magnetic field induced gyration of the material. By comparing the equations (122) and (123)  $g$  can be identified to be

$$g = |g|e^{i\theta} = -\frac{n\omega k_m q_{\text{MO}}^2}{m W_{\text{MO}}(\omega)^2}. \quad (124)$$

**5.2.3. Simplified model (3 degrees of freedom).** Solving equations (116a) and (116b) for  $x_{\text{MO}}$  and  $y_{\text{MO}}$  yields in the limit of a small magnetic influence (i.e.  $k_m \ll W_{\text{MO}}$ )

$$x_{\text{MO}} = + \frac{1}{W_{\text{MO}}(\omega)} (m^{-1} q_{\text{MO}} E_x - k_{\text{MO,TM}} \dot{x}_{\text{TM}}) - ik_m \frac{\omega}{W_{\text{MO}}(\omega)^2} (m^{-1} q_{\text{MO}} E_y - k_{\text{MO,TE}} \dot{y}_{\text{TE}}) \quad (125)$$

$$y_{\text{MO}} = + \frac{1}{W_{\text{MO}}(\omega)} (m^{-1} q_{\text{MO}} E_y - k_{\text{MO,TE}} \dot{y}_{\text{TE}}) + ik_m \frac{\omega}{W_{\text{MO}}(\omega)^2} (m^{-1} q_{\text{MO}} E_x - k_{\text{MO,TM}} \dot{x}_{\text{TM}}). \quad (126)$$

The equations (125) and (126) can now be inserted into the equations (116c) and (116d) to eliminate  $x_{\text{MO}}$  and  $y_{\text{MO}}$ . It is reasonable to assume that the MO oscillator couples only weakly to the far field and that the system is mainly driven via the waveguide and the plasmon oscillators. This means that the driving forces of the MO oscillator are assumed to be much weaker than the internal coupling forces ( $|m^{-1} q_{\text{MO}} E_x| \ll |k_{\text{MO,TM}} \dot{x}_{\text{TM}}|$  and  $|m^{-1} q_{\text{MO}} E_y| \ll |k_{\text{MO,TE}} \dot{y}_{\text{TE}}|$ ). In this limit and by using the equation (124), the equations (116c) and (116d) can be rewritten as

$$\begin{aligned} & (-\omega^2 + \Omega_{\text{TM}}^2 - 2i\Gamma_{\text{TM}}\omega - \frac{k_{\text{MO,TM}}^2}{W_{\text{MO}}(\omega)}) x_{\text{TM}} \\ & - ig \frac{mk_{\text{MO,TM}} k_{\text{MO,TE}}}{nq_{\text{MO}}^2} y_{\text{TE}} - k_{\text{P,TM}} x_{\text{P}} = m^{-1} q_{\text{TM}} E_x \end{aligned} \quad (127)$$

$$\begin{aligned} & (-\omega^2 + \Omega_{\text{TE}}^2 - 2i\Gamma_{\text{TE}}\omega - \frac{k_{\text{MO,TE}}^2}{W_{\text{MO}}(\omega)}) y_{\text{TE}} \\ & + ig \frac{mk_{\text{MO,TM}} k_{\text{MO,TE}}}{nq_{\text{MO}}^2} x_{\text{TM}} = m^{-1} q_{\text{TE}} E_y. \end{aligned} \quad (128)$$

Together with (116e), these two equations fully describe the optical system. Hence, the applied approximations led to a reduction to only three equations. Usually, the material resonance of the MO slab is far away from the excitation frequency and the waveguide frequencies (i.e.  $\Omega_{\text{MO}} \gg \Omega_{\text{TM}}, \Omega_{\text{TE}}, \omega$ ), which means that the terms proportional to  $1/W_{\text{MO}}(\omega)$  can be neglected. This results in the following set of equations:

$$\begin{aligned} (-\omega^2 + \Omega_{\text{TM}}^2 - 2i\Gamma_{\text{TM}\omega})x_{\text{TM}} \\ - ibe^{i\theta}y_{\text{TE}} - k_{\text{P, TM}}x_{\text{P}} \end{aligned} = m^{-1}q_{\text{TM}}E_x \quad (129a)$$

$$\begin{aligned} (-\omega^2 + \Omega_{\text{TE}}^2 - 2i\Gamma_{\text{TE}\omega})y_{\text{TE}} \\ + ibe^{i\theta}x_{\text{TM}} \end{aligned} = m^{-1}q_{\text{TE}}E_y \quad (129b)$$

$$(-\omega^2 + \Omega_{\text{P}}^2 - 2i\Gamma_{\text{P}\omega})x_{\text{P}} \quad (129c)$$

$$-k_{\text{P, TM}}x_{\text{TM}} = m^{-1}q_{\text{P}}E_x, \quad (129d)$$

where  $b = |g|mk_{\text{MO, TM}}k_{\text{MO, TE}}/nq_{\text{MO}}^2$  and  $\theta = \arg(g)$ . For an absent driving electrical field, i.e. for  $\mathbf{E}_{\parallel} = 0$ , the equations above represent a quadratic eigenvalue problem for the eigenfrequencies  $\omega$ . Obtaining these eigenfrequencies would therefore require to find the roots of a polynomial with degree six, which cannot be accomplished analytically. However, by applying a rotating wave approximation, the quadratic eigenvalue problem can be reduced to a linear eigenvalue problem. This approximation is valid, when the relevant frequencies are in the same range, that is

$$-\omega^2 + \Omega_j^2 \approx (\Omega_j - \omega)2\bar{\Omega}, \quad j = \text{TM, TE, P.} \quad (130)$$

Here,  $\bar{\Omega}$  is a constant average frequency close to  $\omega$ ,  $\Omega_{\text{TM}}$  and  $\Omega_{\text{TE}}$ . In this approximation, the equations of motion become

$$(\omega_{\text{TM}} - \omega)x_{\text{TM}} - i\beta e^{i\theta}y_{\text{TE}} - \kappa x_{\text{P}} = \eta\rho_{\text{TM}}E_x \quad (131a)$$

$$(\omega_{\text{TE}} - \omega)y_{\text{TE}} + i\beta e^{i\theta}x_{\text{TM}} = \eta\rho_{\text{TE}}E_y \quad (131b)$$

$$(\omega_{\text{P}} - \omega)x_{\text{P}} - \kappa x_{\text{TM}} = \eta\rho_{\text{P}}E_x. \quad (131c)$$

The following definitions were used:  $\beta = b/2\bar{\Omega}$ ,  $\kappa = k_{\text{P, TM}}/2\bar{\Omega}$ ,  $\eta = 1/2m\bar{\Omega}n$ ,  $\rho_j = nq_j$  and  $\omega_j = \Omega_j - i\Gamma_j$ , with  $j = \text{TM, TE, P}$ , where  $n$  is the oscillator density in the effective medium. When writing the equations above in the matrix form

$$(M_0 + \Delta M - I\omega) \mathbf{x} = \eta R \mathbf{E}_{\parallel}, \quad (132)$$

with  $\mathbf{x} = (x_{\text{TM}}, x_{\text{P}}, y_{\text{TE}})^T$  and

$$M_0 = \begin{pmatrix} \omega_{\text{TM}} & -\kappa & 0 \\ -\kappa & \omega_{\text{P}} & 0 \\ 0 & 0 & \omega_{\text{TE}} \end{pmatrix} \quad (133)$$

$$\Delta M = \beta e^{i\theta} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix} \quad (134)$$

$$R = \begin{pmatrix} \rho_{\text{TM}} & 0 \\ \rho_{\text{P}} & 0 \\ 0 & \rho_{\text{TE}} \end{pmatrix}, \quad (135)$$

it is evident that we arrived at the equation of motion as utilized in section 5.1.

### 5.3. Conclusion

In summary, the dispersion of the MO response of hybrid magnetoplasmonic waveguides can now be understood in the picture of a simple oscillator model including the nonreciprocal Lorentz force. In the case of a weak influence of the Lorentz force, analytical expressions for the optical response were obtained, which confirm previous numerical findings. Importantly, the spectral line shape of the MO response is fully determined by the optical properties of the system for zero magnetic field. Only the overall magnitude of the MO response is determined by the applied magnetic field.

The theory in this section provides the understanding required to further develop hybrid magnetoplasmonic systems in highly integrated optics, demanding actively controlled optical modulation [195], magnetic field sensing [196, 197], refractive index sensing [145], and optical isolation [12–14, 16–18]. It should also be mentioned that the presented findings can not only help to understand and optimize existing sample geometries, but can also be transferred to other geometries, such as 2D plasmonic gratings. Furthermore, by removing all plasmonic oscillators, the case of a purely dielectric grating-waveguide combination can be realized.

In section 6 the presented model will be applied to a system consisting of an EuS thin film and an embedded plasmonic grating. By means of the model, it will be shown, that in the case of weak coupling between the plasmonic and waveguide resonance, the classical optical analog of electromagnetically induced transparency and absorption can be realized. Moreover, the model will be of fundamental importance to understand why the corresponding magneto-optic response in such a system is so exceptionally large.

In respect of non-linear magnetoplasmonics, the presented model could also be of fundamental relevance. Although the described model is fully linear, a non-linear extension would be straight-forward by adding higher order coupling terms [210].

## 6. Giant Faraday effect via induced absorption

In this section a new hybrid magnetoplasmonic thin film structure will be demonstrated, that resembles the classical optical analog of electromagnetically induced absorption. In transmission geometry the gold nanostructure embedded in an EuS film induces giant Faraday rotation of up to  $14^\circ$  for a thickness of less than 200 nm and a magnetic field of 5 T at  $T = 20$  K. By varying the magnetic field from  $-5$  T to  $+5$  T, a rotation tuning range of over  $25^\circ$  is realized. As this is only a factor of three away from the Faraday isolation requirement, the demonstrated concept could lead to highly integrated, nonreciprocal photonic devices for optical isolation, light modulation, and optical magnetic field sensing.

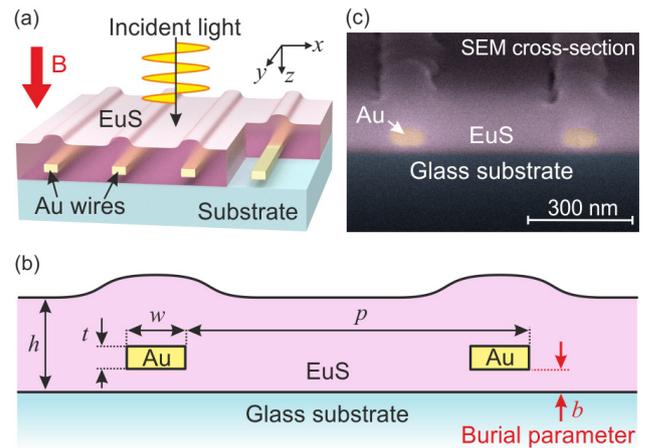
This section has been adapted with permission from the dissertation [43] and the article [42] published under the Creative Commons Attribution 4.0 International license.

### 6.1. Realization with EuS-Au hybrid systems

In the last two sections it was demonstrated how the magneto-optic (MO) response of a dielectric thin film is enhanced by the attachment of a plasmonic grating. This technique also allows the amplified MO response to be spectrally tailored by tuning the grating parameters. However, the MO performance of such a system is limited by the low  $Q$  factor of the plasmon resonances in the grating. In the following, it will be shown that this limitation can be lifted elegantly by a novel approach, which is based on a classical optical analog of electromagnetically induced absorption (EIA) [181, 182]. Here, a strongly damped plasmon oscillation is weakly coupled to a narrow linewidth waveguide resonance with a phase delay, leading to constructive interference. By tuning this coupling carefully, a high- $Q$  absorptive hybrid mode is realized, which can be used to resonantly amplify the Faraday rotation response. Furthermore, the EIA mechanism allows to utilize the high oscillator strength of the plasmonic resonance, leading to an efficient coupling of the incident light into the structure without reducing the effective  $Q$  factor due to the broad plasmonic resonance as was the case in previous approaches [34, 36, 40]. Although being only less than 200 nm thick, the presented novel structure design exhibits an order of magnitude better rotation capability than previous approaches that only resulted in fractions of degree rotation [24–26, 36, 63].

Furthermore, for low magnetic fields, the presented technique still yields rotation angles that were only achievable with 20 times stronger field strength in previous approaches [40]. The performance of the thin film structure is only a factor of 3 away from the  $45^\circ$  that are required for building a Faraday isolator which usually requires MO media with a thickness on the order of centimeters. The first demonstration of plasmonic resemblances of EIA [211, 212] and its related effect, namely electromagnetically induced transparency (EIT) [213–215], triggered significant attention. The special dispersion properties of such systems are known to facilitate plasmonic sensing with narrow linewidths [216–218] as well as slow light [219–221] and delay lines [222, 223]. Furthermore, there have been extensive studies on enhanced nonlinear response [224, 225] in such systems. Here, it will be demonstrated that the EIA-like optical dispersion in the presented system also facilitates a giant MO response. While in EIT the light within a narrow spectral band passes through the sample, in the case of EIA real material polarization is excited within a narrow spectral region.

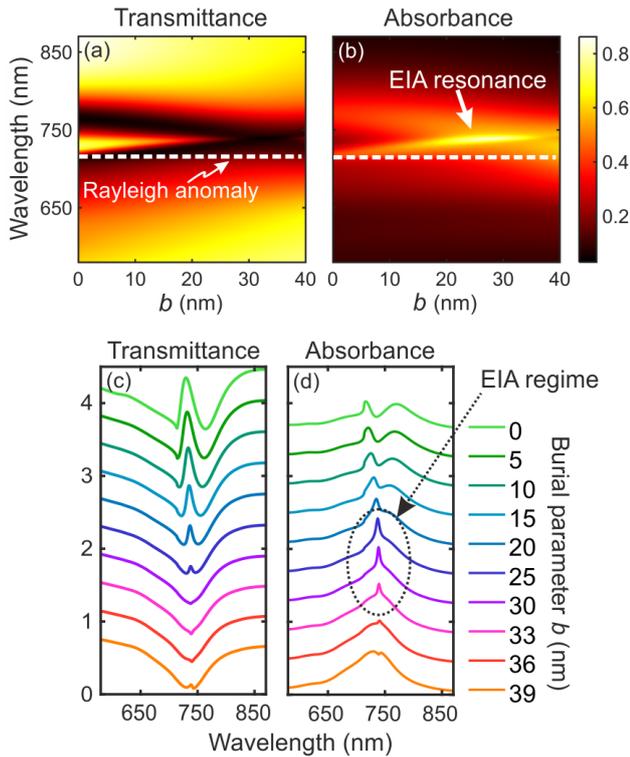
Another important aspect of the presented approach is the introduction of EuS as a new dielectric material for hybrid magnetoplasmonics. At low temperatures, EuS possesses an exceptionally large Verdet constant in combination with a high saturation magnetic flux density, resulting in a potentially very strong MO response [58, 194]. Furthermore, it is transparent in the visible, qualifying it for the utilization in Faraday geometry. While EuS possesses as similar refractive index and absorption coefficient as EuSe [58, 194], EuS is significantly cheaper and already exhibits a stronger MO response at low magnetic field strengths. Further information about the temperature dependence of the EuS material



**Figure 36.** (a),(b) Schematic drawing of the sample geometry.  $p$ , nanowire period;  $t$ ,  $w$  gold nanowire thickness and width. The wires are buried with a distance  $b$  between the glass substrate and their lower edge. The nominal thickness of the EuS magneto-optical waveguide is  $h$ , which is increased near the position of the gold nanowires. (c) Colorized scanning electron micrograph of the sample cross section. The samples are measured at  $T = 20$  K. Reproduced from [42]. CC BY 4.0.

parameters can be found in the supplemental material of [42] as well as in the [53, 58, 194, 226]. We note in passing that in the utilized temperature regime the changes in the optical constants of both gold [227] and the fused silica substrate [228] can be neglected. Despite the need for low temperatures, EuS is well suited for studying plasmonic MO model systems, as it combines the above-mentioned benefits with simple fabrication by physical vapor deposition. This allows for the realization of complex hybrid magnetoplasmonic structure geometries, which are challenging when fabricated with commonly used magneto-optic materials, such as bismuth iron garnet [60] or yttrium iron garnet [229]. The reason is that the deposition of garnet films is a sophisticated process that usually involves pulsed laser deposition and high-temperature annealing that could damage any underlying plasmonic structures.

The utilized structure geometry is depicted in the figures 36(a) and (b). It consists of an EuS thin film with an embedded Au nanowire grating. The EuS film is magnetized by an external magnetic field in the  $z$  direction, which is also the direction of propagation of the incident light. The structure is fabricated in a three-step process. First, the bottom layer of the EuS film with thickness  $b$  is evaporated onto the glass substrate by physical vapor deposition. After that, the gold wire grating with thickness  $t$ , width  $w$ , and period  $p$  is structured via electron beam lithography. In the last step, an EuS film with thickness  $(h - b)$  is evaporated. This results in an EuS-Au thin film structure with a slightly corrugated upper surface. Figure 36(c) shows a colored scanning electron micrograph of the cross section of such a structure with geometry parameters  $p = 490$  nm,  $t = 33$  nm,  $w = 85$  nm,  $h = 139$  nm, and  $b = 33$  nm. For the case of  $b = 0$ , the non-MO dispersion properties of such a metal-dielectric hybrid structure have previously been analyzed in detail by Zentgraf *et al* [230]. Because of the presence of the glass substrate



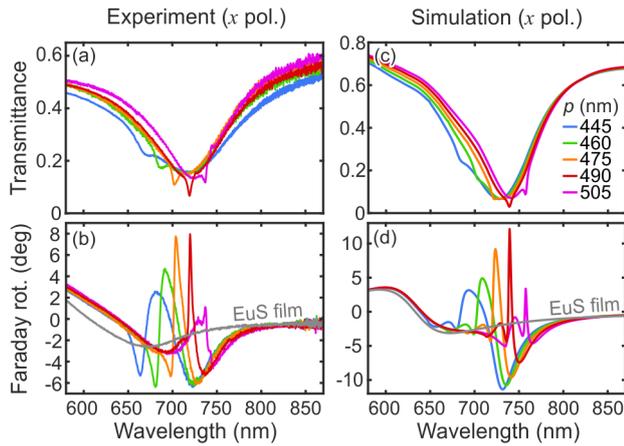
**Figure 37.** (a),(b) Simulated transmittance and absorbance ( $1 - T - R$  with  $T$  and  $R$  denoting transmittance and reflectance, respectively) for  $x$ -polarized incident light and different burial parameters  $b$ . For increasing  $b$  there is a gradual transition from the regime of induced transparency to induced absorption. The white dotted line indicates the Rayleigh anomaly. (c),(d) Slice cuts for a clearer view on the line shapes of the spectra. The oval indicates the EIA regime, characterized by very sharp induced absorption peaks. Reproduced from [42]. CC BY 4.0.

with a higher refractive index than the air above the sample, the  $x$  component of the electric field of the TM waveguide mode is concentrated in the upper part of the EuS waveguide. Hence, the coupling between plasmon and TM waveguide mode is weak in comparison to the case where the metal wires are attached on top. In the work by Zentgraf *et al* it was also demonstrated that this weak coupling can lead to the classical analog of the quantum mechanical effect of electromagnetically induced transparency, resulting in a narrow transmission peak on top of a broad transmission dip for  $x$ -polarized incident light.

**6.1.1. Exploiting the regime of induced absorption.** It will now be demonstrated that by increasing the burial parameter  $b$  in the hybrid system, the phase between the plasmonic mode and the TM waveguide mode can be tuned such that the system undergoes a transition from EIT-like to EIA-like behavior [211, 212]. Figure 37 displays the simulated transmittance and absorbance spectra for  $x$ -polarized incident light and different burial parameters  $b$ . The simulations have been carried out using the Fourier modal method for anisotropic materials [231]. The implementation is based on a scattering matrix algorithm [72], which has been improved by adaptive spatial resolution [208] to enable an efficient derivation of optical properties of metallodielectric systems. The refractive

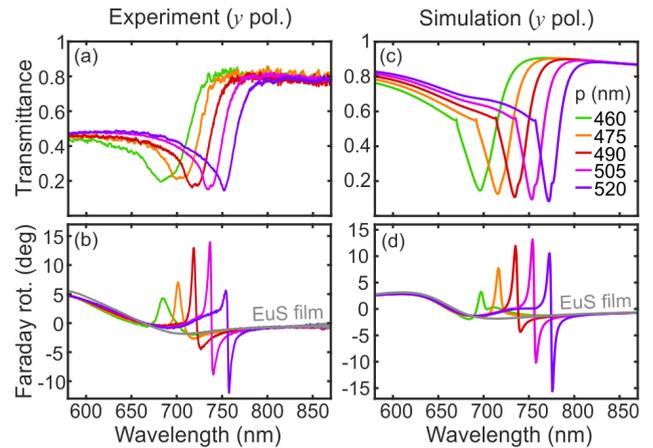
index of the substrate is assumed to be 1.456, and for the gold wires the permittivity data by Johnson and Christy [193] are used. In the investigated wavelength range, the refractive index of EuS is approximately 2. The utilized material model parameters for EuS can be found in the supplemental material of [42]. The geometry parameters were set to  $p = 490$  nm,  $t = 33$  nm,  $w = 85$  nm, and  $h = 139$  nm. In all simulations the bumps in the EuS film above the Au wires are approximated by a rectangular shape with thickness  $t$  and a width of  $2w$ . As expected [230], for small values of  $b$ , the transmittance spectrum exhibits the characteristic EIT line shape with a broad, mainly plasmon-induced dip and a narrow, mainly TM waveguide-induced peak. For increasing values of  $b$ , this effect is reversed and the sharp waveguide feature flips around, resulting in a sharp absorbance peak that is characteristic for an EIA-like system. At 715 nm, the spectra are slightly distorted due to the Rayleigh anomaly [38]. The transition of the spectra with increasing  $b$  can be understood in the picture of coupled Lorentz oscillators introduced by Taubert *et al* [212]: here, a strongly damped Lorentz oscillator (in this case the plasmon) and a less damped Lorentz oscillator (in this case the TM waveguide mode) are coupled via a complex coupling constant that includes an additional phase delay. It was shown that by changing the phase of the coupling constant, the optical response of the system turns from EIT-like to an EIA-like behavior. In the present case, this behavior is particularly pronounced for  $b$  between 25 and 36 nm (see figure 37(d), oval area). Furthermore, by changing the distance between the wire grating and the substrate, the coupling phase can be changed. As in other cases of plasmonic EIT and EIA, the reflection behavior of the system also changes with the coupling phase, which leads to the situation that the EIT coupling regime can be observed most pronounced in the transmittance spectrum, whereas the EIA case can be identified best in the absorbance spectrum [212]. In section 6.2, there will be an extended discussion on interpreting the presented magnetoplasmonic system in the picture of coupled oscillators.

For a burial parameter of  $b = 33$  nm, the EIA coupling regime manifests itself also in the transmittance spectrum in the form of a small dip (see figure 37(c)). For this case the relation between the EIA-like behavior and the MO response was investigated experimentally. The figures 38(a) and (b) display the measured transmittance and Faraday rotation spectra for  $x$ -polarized incident light and different grating periods  $p$ . The measurements are performed at 20 K and a magnetic field of 5 T in the  $z$  direction. The Faraday rotation was measured with a rotating analyzer setup where the incident polarization state was prepared using a polarizer inside the cryostat. For a period of 490 nm, the sharp EIA feature is best centered with respect to the broad, mainly plasmonic, transmittance feature. For the same grating period, at the spectral position of the EIA resonance, the Faraday rotation exhibits a sharp maximum of about  $8^\circ$ , which is a substantial rotation enhancement compared to a bare EuS film with thickness  $h = 139$  nm (gray curve). This behavior can be understood in the following way: For wavelengths close to the narrow EIA resonance, the system acts as a resonator



**Figure 38.** The left-hand column displays the measured transmittance (a) and Faraday rotation (b) for  $x$ -polarized incident light for a wire width of 85 nm, burial parameter  $b = 33$  nm, and different grating periods. The experimental data agree with the performed simulation shown in the right-hand column (c),(d). Reproduced from [42]. CC BY 4.0.

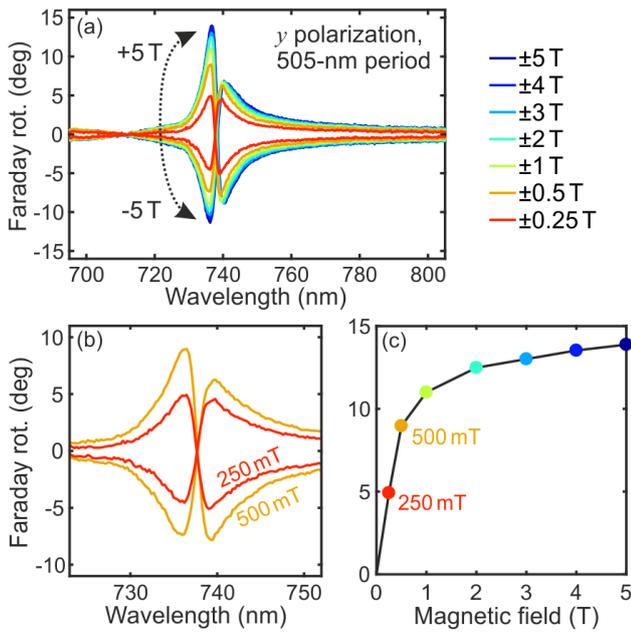
with a relatively high  $Q$  factor, implying multiple vertical round-trips of the light through the MO material before it is coupled out in  $z$  direction. Due to Lorentz non-reciprocity, the polarization rotation adds up for each propagation cycle, resulting in an enhanced rotation compared to a single pass through the EuS film. Furthermore, the EIA mechanism allows utilizing the high oscillator strength of the plasmonic resonance, leading to an efficient coupling of the incident light into the structure without reducing the effective  $Q$  factor due to the broad plasmonic resonance as was the case in previous approaches [34, 36, 40]. As such, the plasmonic EIA mechanism provides an elegant way to greatly increase the interaction between the incident light and the MO material. In section 6.2 the connection between the EIA dispersion and Faraday rotation enhancement will be discussed in further detail by means of a coupled oscillator model. The measurement is also in good agreement with the full numerical simulation displayed in the figures 37(c) and (d). In the measured wavelength region, the Faraday rotation of EuS shows a notable wavelength dependence due to a material resonance (see gray line). Hence, in order to obtain realistic simulation results, the material dispersion of EuS is modeled with a MO Lorentz oscillator (see section 2.3.1). The free model parameters are fitted such that the measured MO response of a bare EuS film matches the simulated MO response in the wavelength range of interest between 650 and 850 nm. The discrepancy between the measured and simulated Faraday rotation below 650 nm is due to the limited model accuracy in this range. In the transmittance spectra, the grating-induced Rayleigh anomaly is barely visible. For example, for  $p = 490$  nm it occurs at 715 nm and is most visible in the simulated Faraday rotation, where it manifests itself as a small kink. However, the Rayleigh anomaly has no major influence on the Faraday rotation enhancement as it is well separated from the enhancement by the EIA resonance.



**Figure 39.** The left-hand column displays the measured transmittance (a) and Faraday rotation (b) for  $y$ -polarized incident light for a wire width of 75 nm, burial parameter  $b = 33$  nm, and different grating periods  $p$ . The experimental data agree well with the performed simulation shown in the right-hand column (c),(d). For a period of 505 nm, the Faraday rotation reaches values of up to  $14^\circ$ . Reproduced from [42]. CC BY 4.0.

Also for  $y$ -polarized incident light, large Faraday rotation angles can be realized. This is illustrated by the figures 39(a) and (b), which depict the corresponding transmittance and Faraday rotation spectra for a sample geometry with  $t = 33$  nm,  $w = 75$  nm,  $h = 139$  nm,  $b = 33$  nm, and different grating periods  $p$ . Since a direct excitation of the plasmon resonance and TM waveguide resonance is only possible for  $x$ -polarized light, they have no discernable influence on the transmittance line shape. However, as the  $x$ - and  $y$ -polarized eigenmodes are coupled via the magnetic field, the Faraday rotation spectrum for  $y$ -polarized incidence is strongly influenced by the  $x$ -polarized eigenmodes (and vice versa) [41], which results in strongly enhanced Faraday rotation also for  $y$ -polarized incidence. Furthermore, this behavior will be discussed in more detail in section 6.2. For a grating period of 505 nm, the Faraday rotation reaches values of up to  $14^\circ$  at a transmittance value of 17%. Given the thin structure profile of below 200 nm, this is a giant Faraday rotation value and only a factor of 3 away from a Faraday isolator. Again, the measurement results are in good agreement with the numerical simulations plotted in the figures 39(c) and (d), except that the Rayleigh anomalies (kinks in the left shoulders of the transmittance spectra) are washed out in the measurement. This happens certainly due to sample roughness and limited fabrication accuracy. As in the case of  $x$ -polarized incidence, the Rayleigh anomalies are well separated from the Faraday rotation enhancement feature and have no significant influence on the Faraday rotation spectra.

**6.1.2. Influence of magnetic field strength.** Furthermore, since the rotation angle scales with the applied magnetic field, the presented structure offers an impressive polarization tuning range of over  $25^\circ$ . This is indicated in figure 40(a), where the Faraday rotation spectra are plotted for different magnetic

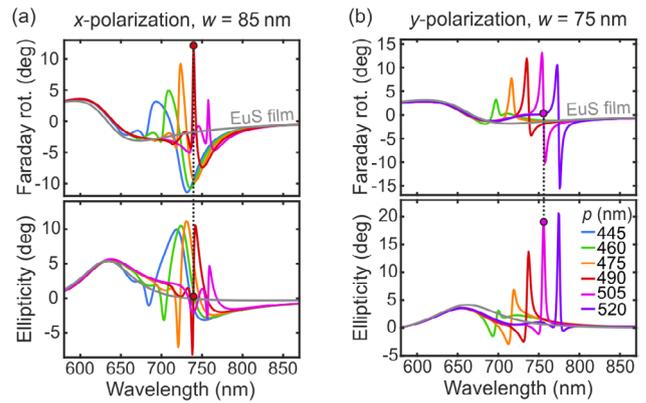


**Figure 40.** (a) Measurement of the magnetic field dependence of the polarization rotation for a period of 505 nm. (b) Closer view on the rotation spectra for weak magnetic fields. Already for 250 mT the Faraday rotation reaches values of over  $4^\circ$ . (c) Saturation behavior of the Faraday rotation at 737 nm. Reproduced from [42]. CC BY 4.0.

field strengths ranging from  $-5$  to  $+5$  T. The spectra for inverted magnetic fields exhibit almost perfectly mirror symmetric shapes. Any deviations from absolute mirror symmetry can be explained by a slight misalignment between the gold wires and the incident polarization. Figure 40(b) shows a close-up of the Faraday rotation enhancement feature for lower magnetic field strengths that can be realized easily using standard permanent magnets [232]<sup>2</sup>. Even for magnetic fields as low as 250 mT, the Faraday rotation still reaches values of up to  $4.9^\circ$ . This is a similar rotation performance as was previously realized for a 220 nm thick hybrid structure based on EuSe, however, only with 20 times larger magnetic fields [40]. Figure 40(c) displays the Faraday rotation at 737 nm as a function of applied magnetic field and illustrates the saturation behavior. Up to 500 mT the rotation angle increases linearly with the applied magnetic field, whereas for larger fields saturation sets in and the response curve flattens out.

**6.1.3. Simulation of the ellipticity spectra.** As the utilized measurement setup does not allow for measuring the ellipticity spectra, in the following, the numerically simulated ellipticity spectra are provided. Figure 41 displays the ellipticity spectra of the sample geometries with 85 and 75 nm wire width. In the first case, a grating period of 490 nm results in the largest Faraday rotation peak. At its center wavelength, the corresponding ellipticity is close to zero. This behavior is consistent with the following rule of thumb, originating from

<sup>2</sup> Due to the  $\omega$  based mathematical structure of the model, the response curves were plotted over angular frequency. This should be kept in mind when the modeled spectra are compared to the measured spectra, which are plotted over wavelength.



**Figure 41.** Simulated Faraday rotation and ellipticity spectra corresponding to the sample geometries with 85 nm wire width (a) and 75 nm wire width (b). Reproduced from [42]. CC BY 4.0.

the Kramers–Kronig relations [44]: a symmetric Faraday rotation peak usually corresponds to asymmetric (s-shaped) ellipticity spectrum with a zero-crossing at the center wavelength of the rotation peak. On the other hand, a symmetric ellipticity peak corresponds to s-shaped Faraday rotation spectrum with the zero crossing at the center wavelength of the ellipticity peak. For example, the latter behavior is exhibited for a wire width of 75 nm, and a period of 505 nm, as illustrated in figure 41(b).

## 6.2. Harmonic oscillator modeling

To obtain further insight into the connection between EIA-like modal coupling and the enhancement of the magneto-optic response, it is useful to view the presented magnetoplasmonic system in the picture of coupled oscillators, as depicted in figure 42(a). This oscillator scheme was originally introduced in [41] and is now modified in analogy to the non-magneto-optic model by Taubert *et al* to account for EIA [212]. In this very simple model, the magnetoplasmonic system is described using two charged oscillators that are driven by the external light field: the oscillator on the right-hand side represents the plasmonic excitation, and its movement is restricted to the  $x$  direction, as the plasmonic resonance can only be excited for  $x$ -polarized light. The second oscillator is associated with the waveguide excitations inside the MO slab. This oscillator can be displaced both in the  $x$  and  $y$  direction, which corresponds to the excitation of TM and TE polarized waveguide modes respectively. As discussed by Taubert *et al*, it is crucial for realizing the coupling regime of EIA that the coupling between the two contributing oscillators includes a retardation phase [211]. Hence, the coupling between plasmon and TM waveguide is modeled with a complex coupling constant  $\tilde{\kappa} = \kappa \exp(i\phi)$ , with  $\phi$  as the phase. The influence of the applied static magnetic field is taken into account via the Lorentz force acting on the 2D waveguide oscillator. The resulting electric susceptibility associated with this oscillator system is given by [41]

$$\chi(\omega) = \begin{pmatrix} \chi_x & +\chi_{xy} \\ -\chi_{xy} & \chi_y \end{pmatrix} = \eta R^T M^{-1} R, \quad (136)$$

where the matrices

$$M = \begin{pmatrix} \Omega_{\text{TM}} & -\kappa e^{i\phi} & -i\beta \\ -\kappa e^{i\phi} & \Omega_{\text{P}} & 0 \\ +i\beta & 0 & \Omega_{\text{TE}} \end{pmatrix} \quad (137)$$

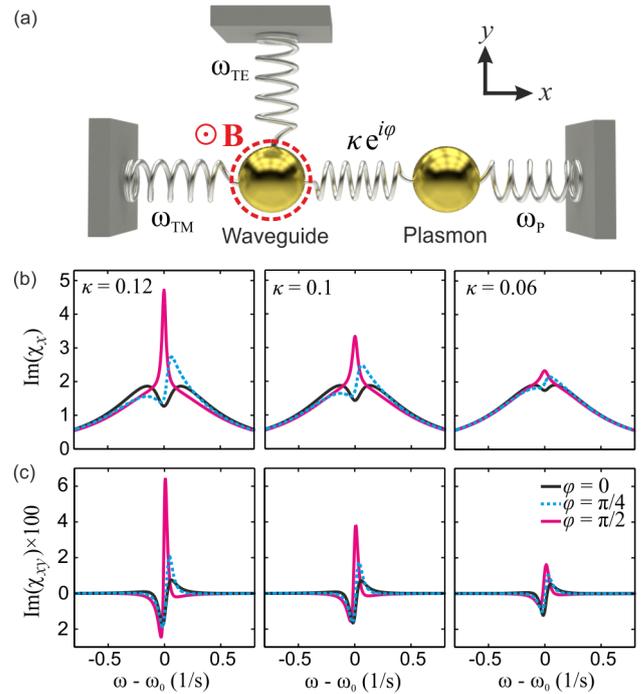
and

$$R = \begin{pmatrix} \rho_{\text{TM}} & 0 \\ \rho_{\text{P}} & 0 \\ 0 & \rho_{\text{TE}} \end{pmatrix} \quad (138)$$

are introduced. Here, the terms  $\Omega_i = \omega_i - \omega - i\gamma_i/2$ , ( $i = \text{TM}, \text{P}, \text{TE}$ ) contain the eigenfrequencies and damping coefficients of the individual oscillators. The magneto-optic coefficient  $\beta$  is proportional to the gyration of the waveguide material and, thus, to the applied magnetic field. The occurrence with two different signs originates from the cross product in the Lorentz force. The nonzero components  $\rho_i$  ( $i = \text{TM}, \text{P}, \text{TE}$ ) in the matrix  $R$  are proportional to the charge of the oscillators and relate to the individual oscillator strengths.  $\eta$  is a proportionality constant.

In the following, this model is used to illustrate the relation between the EIA and EIT coupling regimes as well as their influence on the magneto-optic response. For this, to keep the degrees of freedom to a minimum, the following simple (and dimensionless) parameter constellation is utilized: the individual resonances in the system are spectrally close, while the waveguide modes having significantly smaller linewidths than the plasmon. Hence, the eigenfrequencies of the three oscillators are assumed to be equal, i.e.  $\omega_{\text{TM}} = \omega_{\text{TE}} = \omega_{\text{P}} = \omega_0$  and for the damping coefficients it is assumed that  $\gamma_{\text{P}} = 1$  and  $\gamma_{\text{TM}} = \gamma_{\text{TE}} = 0.1$ . Furthermore, the proportionality constant  $\eta$  is set to 1 and it is assumed that the plasmon oscillator couples much stronger to the external light field than to the TM waveguide mode. Hence, the oscillator strengths are weighted by setting  $\rho_{\text{TM}} = 0$  and  $\rho_{\text{P}} = 1$ . Furthermore, the oscillator strength of the TE waveguide oscillator was set to  $\rho_{\text{TE}} = 0.32$ , which represents the case in which  $\chi_x$  and  $\chi_y$  (not shown here) are approximately equal.

The plots in figure 42(b) show the resulting frequency dependence of the imaginary part of  $\chi_x$ , which is proportional to the absorbance of  $x$ -polarized incident light. The different line colors correspond to the different coupling phases. Furthermore, from the left to the right column, the coupling amplitude  $\kappa$  is decreased. All spectra of  $\chi_x$  show a broad, mainly plasmon-induced, background with a sharp modulation on top. For  $\phi = 0$  this modulation is a narrow dip at  $\omega_0$  and corresponds to the case of EIT. Increasing the coupling phase to  $\phi = \pi/2$  turns this modulation into a narrow EIA resonance, while the case of  $\phi = \pi/4$  represents an intermediate regime. Furthermore, an increase of  $\kappa$  leads to a more pronounced modulation. However, at some point when  $\kappa$  is increased further (not shown here), the coupling is not weak anymore and a significant mode splitting occurs for  $\phi = 0$ , and for  $\phi = \pi/2$  solutions with negative absorption can emerge. As such solutions correspond to the presence of gain, they are not relevant for the passive plasmonic system discussed here [212]. In summary, the classical analogs



**Figure 42.** (a) Oscillator model representing the magneto-plasmonic nanostructure. A precondition for EIA is that the coupling between the TM waveguide mode and the plasmon mode is weak and includes retardation. This is taken into account by the complex coupling constant  $\kappa \exp(i\phi)$  with phase  $\phi$ . (b) Imaginary part of the component  $\chi_x$ , which is proportional to the absorbance of  $x$ -polarized incident light. (c) Imaginary part of  $\chi_{xy}$ , which induces polarization rotation of both  $x$ - and  $y$ -polarized incident light. Reproduced from [42]. CC BY 4.0.

of EIT and EIA are indeed very closely related: both scenarios occur in the regime of weak coupling where there is no significant spectral repulsion between the narrow linewidth mode (in this case, the TM waveguide mode) and the broad linewidth mode (in this case, the plasmon). The only difference between the two scenarios lies in the coupling phase  $\phi$ . It should be pointed out that the absorption behavior is approximately independent of the magnetic field, as  $|\beta|$  can be assumed to be small (here, it is set to 0.001), in which case it predominantly influences the off-diagonal elements of  $\chi$  but not the diagonal elements [41].

The corresponding magneto-optic response is encoded in the off-diagonal elements of  $\chi$  that are proportional to  $\beta$  and, thus, to the magnetic field. In first approximation, the Faraday rotation of the system is proportional to the imaginary part of  $\chi_{xy}$  (see section 2.2.3), which is plotted in figure 42(c). It should be noted that the real part of  $\chi_{xy}$  would correspond to the ellipticity (not plotted here). The graphs nicely illustrate the fundamental working principle of the system: with increasing  $\kappa$ , not only the modulation of  $\chi_x$  increases (i.e. the EIA or EIT resonances become more pronounced), but also the modulation of  $\chi_{xy}$  becomes stronger. In other words, due to the magnetic field, the EIA or EIT resonances not only occur in absorption, but also in Faraday rotation. The more pronounced the EIA or EIT resonances are, the larger the Faraday rotation enhancement becomes. Furthermore, for a constant  $\kappa$  this modulation of

$\chi_{xy}$  is strongest for  $\phi = \pi/2$ , i.e. for the EIA case. At this point it should be noted that in practice the difference in the MO response obtained by EIA-like and EIT-like coupling can be less pronounced than in the modeled spectra. The reason is that tuning the structure geometry from the EIT to the EIA case usually also slightly influences the other coupling parameters such as  $\kappa$ , which also have a strong influence on  $\chi_{xy}$ . However, the model correctly reflects the trend that the EIA case produces stronger MO response than the EIT case. Another revealing aspect of this model is that although only the oscillators moving in the  $x$  direction contribute to the EIA resonance, this resonance also leads to increased polarization rotation for  $y$ -polarized light: this follows from the fact that  $\chi$  is antisymmetric. Hence, the resonance in  $\chi_{xy}$  translates to a conversion from  $x$  to  $y$  polarization as well as to a conversion from  $y$  to  $x$  polarization. However, in general, the Faraday rotation spectra for  $x$ - and  $y$ -polarized light are not completely equal since for finite propagation distances also the diagonal components  $\chi_x$  and  $\chi_y$  possess an influence [41, 44]. As shown by the measurements in figure 39, the Faraday rotation can be even larger for  $y$ -polarized light than for  $x$ -polarized light. It should also be pointed out that the line shape of  $\chi_{xy}$  depends on the complex phase of the magneto optical coupling constant  $\beta$ , which is related to the gyration of the MO material [41]. For the present example, this phase is set to  $-45^\circ$ . Changing this phase does not affect the average magnitude of the magneto-optic response, but only determines whether the  $\text{Im}(\chi_{xy})$  spectra show a down-up line shape (negative phase), an up-down line shape (positive phase), or a peak (zero phase) when plotted over angular frequency. Due to the  $\omega$  based mathematical structure of the model, the response curves were plotted over angular frequency. This should be kept in mind when the modeled spectra are compared to the measured spectra, which are plotted over wavelength.

As a final remark, it should be mentioned that the presented oscillator model is the simplest possible approach to illustrate the relation between EIA-like coupling and the enhanced MO response in the system. Of course, this results in neglecting secondary influences observed in the actual measurements, such as material dispersion, background Faraday rotation due to the film, or diffraction effects such as Rayleigh anomalies [41]. Nevertheless, the benefit of this model is that it provides a very intuitive description of the system.

### 6.3. Conclusion

The design flexibility enabled by the use of EuS as magneto-optic material was exploited to realize a hybrid magnetoplasmonic thin film structure that represents the classical optical analog of EIA. Unlike in previous approaches where the low quality factor of localized surface plasmon resonances limited the Faraday rotation enhancement, the coupling regime of EIA allows us to leverage both the high quality factor of the waveguide resonances and the large oscillator strength of the plasmons. Both these aspects result in dramatically increased light-matter interaction and MO response. As a result, the

EIA-like system induces giant Faraday rotation of over  $14^\circ$  for a thickness of less than 200 nm and a magnetic field of 5 T at  $T = 20$  K. By varying the magnetic field from  $-5$  T to  $+5$  T, an impressive rotation tuning range of over  $25^\circ$  is realized. This is a MO performance that exceeds any previous approach considerably.

## 7. Conclusion and outlook

We have seen that the field of nonreciprocal plasmonics has become big and multifaceted. Furthermore, every method of breaking Lorentz reciprocity usually comes with certain practical limitations, such as fabrication difficulties, optical nonlinearities, size constraints and overall technical feasibility. The various approaches in nonreciprocal nanophotonics can be divided into two groups: firstly, there have been efforts to realize enhanced MO effects of nanostructures with the aim to eventually integrate them into miniaturized versions of conventional nonreciprocal devices such as Faraday isolators. Secondly, there have been several attempts to directly realize complete nonreciprocal photonic devices such as optical isolators and circulators.

Our analysis of the second group of approaches showed that, so far, there has not been realized a system with practical degrees of isolation and low losses simultaneously. Especially nonlinear systems seem inherently hampered as nonlinear effects are usually exploited through losses. Furthermore, the intensity dependent optical isolation is usually not practical. Time modulated systems are very promising, as they already reached practical relevance within the microwave regime. However, optical realizations are usually only theoretical or proof-of-concept realizations [8]. Nevertheless, the field still becomes more and more active.

With regards to enhanced Faraday rotation we have seen that dielectric MO films in combination with plasmonic enhancement have been proven to be a very powerful approach. In comparison, ferromagnetic nanoantennas are hampered by large optical losses and significantly lower MO response due to low quality resonances. However, magnetoplasmonic antennas remain a very intriguing platform for sensing applications [145]. Furthermore, as the required fabrication techniques are widely available, magnetoplasmonic antennas are also attractive for studying fundamental MO interaction effects such as chirality in combination with magnetic gyration. For such studies it is beneficial that magnetoplasmonic antennas comprise a rather simple and clean physical system.

In this review we demonstrated that especially large Faraday rotation can be achieved for hybrid magnetoplasmonic crystals employing one of the two working principles: In the first case, a gold nanowire grating is attached on top of a magneto-optic (MO) thin film. The strong coupling between the plasmon resonance in the gold grating and a waveguide mode in the MO film results in the formation of a waveguide-plasmon-polariton (WPP). This hybrid mode can facilitate large Faraday rotation of up to  $4^\circ$  for a 220 nm thick EuSe-Au structure. Furthermore, the WPP dispersion properties can be leveraged to tailor the working wavelength of a Faraday rotator via the

grating parameters. However, even stronger MO response can be obtained for the second type: here, a plasmonic gold grating is embedded into a MO film. Such a system can be regarded as a classical optical analog of electromagnetically induced absorption (EIA). Its unique dispersion properties allow to compensate for the low  $Q$  factor of the plasmonic resonance, which limits the MO response in the WPP-based approach. The presented EIA-like EuS-Au structures induce giant Faraday rotation of up to  $14^\circ$  for a thickness of less than 200 nm and a magnetic field of 5 T at  $T = 20$  K. Furthermore, by varying the magnetic field from  $-5$  to  $+5$  T, the polarization of the transmitted light can be continuously tuned over an impressively wide range of over  $25^\circ$ . As the realized magneto-optic response is only a factor of three away from the Faraday isolation requirement, this concept could lead to highly integrated, nonreciprocal photonic devices for optical isolation, light modulation, and optical magnetic field sensing.

Furthermore, as was demonstrated recently, strong MO dispersion of nanostructured systems can also be leveraged for refractive index sensing, allowing very precise detection of biochemical substances [145]. Here, the exceptionally sharp spectral features in the MO response of EIA systems could also turn out to be a powerful tool.

A fruitful strategy to even further increase the MO response of the EuS-Au structures could be to use a thicker MO slab in combination with gratings that possess a similar lattice constant as in the case of the EIA structures discussed in section 6. This way, higher-order waveguide resonances could be excited, which, in combination with EIA-like dispersion, could lead to Faraday rotation values closer to or even above  $45^\circ$ . Also some sort of physical or chemical treatment to enhance the surface quality of the EuS film is expected to further increase the resonator performance resulting in larger Faraday rotation.

In future designs, the necessity for low temperatures could be significantly relaxed by doping europium compounds with gadolinium. For example, it has been shown that doping EuO with gadolinium can raise the Curie temperature (where the Faraday rotation starts to drop) from 69 to 135 K [57, 205], which is well above liquid nitrogen temperature. Of course, the concept of WPP- and EIA-boosted MO response is not restricted to Eu compounds and can in principle be applied to many materials. The only requirements are a sufficient transparency of the MO material and a suitable procedure for the nanostructure fabrication. For example, although BIG only provides moderate MO response, it allows room-temperature operation and exhibits high transparency in the near-infrared wavelength range.

EuS, on the other hand, enables simple fabrication of complex layer-based magneto-optic geometries. This is a very powerful and rare property among transparent magneto-optic materials. Since EuS is much cheaper than EuSe and also provides stronger MO response already at low magnetic fields, EuS could become a trigger for other interesting and potentially very powerful magneto-optic and magnetoplasmonic designs. This could include both 2D and 3D systems, for instance EuS photonic crystals, 2D nanopatches, as well as even magneto-optic metamaterials [20, 233].

The theory based on harmonic oscillators allows the description of the optical response of hybrid magnetoplasmonic systems in a simple and intuitive picture. This model provides analytical expressions for the optical response and confirms previous numerical and experimental findings. One important outcome of the model is that the spectral line shape of the MO response is fully determined by the optical properties of the system for zero magnetic field. Only the overall magnitude of the MO response is determined by the applied magnetic field. An extension of the oscillator model to 2D and 3D systems should be feasible. This could facilitate the development and optimization of magnetoplasmonic crystals of higher dimensions. Furthermore, by removing all plasmonic oscillators, the case of a purely dielectric grating-waveguide combination can be modeled. In prospect of non-linear magnetoplasmonics, the presented model could also be of fundamental relevance. Although the model is fully linear, a non-linear extension would be straight-forward by adding higher order coupling terms [210].

Furthermore, as for most applications low insertion losses are crucial, the concept of EIT [215] could be a powerful mechanism to realize highly transparent magneto-optical nanophotonic systems. It would be especially interesting to see an experimental realization of magneto-optic EIT metasurfaces as theoretically demonstrated by Christofi *et al* [122] based on idealized material parameters.

Parts of this concluding section have been adapted with permission from the dissertation [43].

## Acknowledgments

The authors acknowledge support from DFG (SPP 1839), BMBF (13N12443 Faraday), MWK Baden-Württemberg (Zukunftsoffensive IV), ERC Complexplas and Baden-Württemberg Stiftung: Spitzenforschung II; Deutsche Forschungsgemeinschaft (DFG): Mercator-Fellowship.

## ORCID iDs

Harald Giessen  <https://orcid.org/0000-0002-4270-3850>

## References

- [1] Landau L and Lifshitz E 1960 *Electrodynamics of Continuous Media* (Oxford: Pergamon)
- [2] Schatz P N and McCaffery A J 1969 The Faraday effect *Q. Rev. Chem. Soc.* **23** 552
- [3] Qui Z Q and Bader S D 2000 Surface magneto-optic Kerr effect *Rev. Sci. Instrum.* **71** 1243
- [4] de Hoop A T 1960 A reciprocity theorem for the electromagnetic field scattered by an obstacle *Appl. Sci. Res. B* **8** 135–40
- [5] Jalas D *et al* 2013 What is-and what is not-an optical isolator *Nat. Photon.* **7** 579–82
- [6] Tinoco I 2007 Theoretical Aspects of Optical Activity Part Two: Polymers (*Advances in Chemical Physics*) (New York: Wiley) pp 113–60

- [7] Shi Y, Yu Z and Fan S 2015 Limitations of nonlinear optical isolators due to dynamic reciprocity *Nat. Photon.* **9** 388–92
- [8] Sounas D L and Alù A 2017 Non-reciprocal photonics based on time modulation *Nat. Photon.* **11** 774
- [9] Iwamura H, Hayashi S and Iwasaki H 1978 A compact optical isolator using a Y3FeO12 crystal for near infra-red radiation *Opt. Quantum Electron.* **10** 393–8
- [10] Shiraishi K, Sugaya S and Kawakami S 1984 Fiber Faraday rotator *Appl. Opt.* **23** 1103
- [11] Gauthier D J, Narum P and Boyd R W 1986 Faraday isolator *Opt. Lett.* **11** 623–5
- [12] Shirasaki M and Asama K 1982 Compact optical isolator for fibers using birefringent wedges *Appl. Opt.* **21** 4296–9
- [13] Bi L, Hu J, Jiang P and Kim D 2011 On-chip optical isolation in monolithically integrated non-reciprocal optical resonators *Nat. Photon.* **5** 758–62
- [14] Fan L, Wang J, Varghese L T, Shen H, Niu B, Xuan Y, Weiner A M and Qi M 2012 An all-silicon passive optical diode *Science* **335** 447–50
- [15] Tsakmakidis K 2013 Non-reciprocal plasmonics *Nat. Mater.* **12** 378
- [16] Ikushima I and Maeda M 1978 Self-coupled phenomena of semiconductor lasers caused by an optical fiber *IEEE J. Quantum Electron.* **14** 331–2
- [17] Morikawa T, Mitsunashi Y, Shimada J and Kojima Y 1976 Return-beam-induced oscillations in self-coupled semiconductor lasers *Electron. Lett.* **12** 435
- [18] Petermann K 1995 External optical feedback phenomena in semiconductor lasers *IEEE J. Sel. Top. Quantum Electron.* **1** 480–9
- [19] Liu M and Zhang X 2013 Plasmon-boosted magneto-optics *Nat. Photon.* **7** 429–30
- [20] Koenderink A F, Alù A and Polman A 2015 Nanophotonics: shrinking light-based technology *Science* **348** 516
- [21] Jain P K, Xiao Y, Walsworth R and Cohen A E 2009 Surface plasmon resonance enhanced magneto-optics (SuPREMO): Faraday rotation enhancement in gold-coated iron oxide nanocrystals *Nano Lett.* **9** 1644–50
- [22] Steel M J, Levy M and Osgood R M 2000 High transmission enhanced Faraday rotation in one-dimensional photonic crystals with defects *IEEE Photonics Technol. Lett.* **12** 1171–3
- [23] Fedyanin A A, Aktsipetrov O A, Kobayashi D, Nishimura K, Uchida H and Inoue M 2004 Enhanced Faraday and nonlinear magneto-optical Kerr effects in magnetophotonic crystals *J. Magn. Magn. Mater.* **282** 256–9
- [24] Diwekar M, Kamaev V, Shi J and Vardeny Z V 2004 Optical and magneto-optical studies of two-dimensional metallodielectric photonic crystals on cobalt films *Appl. Phys. Lett.* **84** 3112–4
- [25] Caballero B, García-Martín A and Cuevas J C 2015 Faraday effect in hybrid magneto-plasmonic photonic crystals *Opt. Express* **23** 22238
- [26] Caballero B, García-Martín A and Cuevas J C 2016 Hybrid magnetoplasmonic crystals boost the performance of nanohole arrays as plasmonic sensors *ACS Photonics* **3** 203–8
- [27] Rosenberg R, Rubinstein C B and Herriott D R 1964 Resonant optical faraday rotator *Appl. Opt.* **3** 1079–83
- [28] Inoue M, Arai K, Fujii T and Abe M 1999 One-dimensional magnetophotonic crystals *J. Appl. Phys.* **85** 5768
- [29] Khartsev S I and Grishin A M 2007 High performance [Bi3Fe5O12/Sm3Ga5O12]m magneto-optical photonic crystals *J. Appl. Phys.* **101** 053906
- [30] Sepúlveda B, González-Díaz J B, García-Martín A, Lechuga L M and Armelles G 2010 Plasmon-induced magneto-optical activity in nanosized gold disks *Phys. Rev. Lett.* **104** 147401
- [31] González-Díaz J B, Sepúlveda B, García-Martín A and Armelles G 2010 Cobalt dependence of the magneto-optical response in magnetoplasmonic nanodisks *Appl. Phys. Lett.* **97** 043114
- [32] Banthí J C, Meneses-Rodríguez D, García F, González M U, García-Martín A, Cebollada A and Armelles G 2012 High magneto-optical activity and low optical losses in metal-dielectric Au/Co/Au-SiO(2) magnetoplasmonic nanodisks *Adv. Mater.* **24** OP36
- [33] Kataja M, Hakala T K, Julku A, Huttunen M J, van Dijken S and Törmä P 2015 Surface lattice resonances and magneto-optical response in magnetic nanoparticle arrays *Nat. Commun.* **6** 7072
- [34] Kreilkamp L E, Belotelov V I, Chin J Y, Neutzner S, Dregely D, Wehler T, Akimov I A, Manfred B, Bernd S and Harald G 2013 Waveguide-plasmon polaritons enhance transverse magneto-optical Kerr effect *Phys. Rev. X* **3** 041019
- [35] Marinchio H, Carminati R, García-Martín A and Sáenz J J 2014 Magneto-optical kerr effect in resonant subwavelength nanowire gratings *New J. Phys.* **16** 015007
- [36] Chin J Y, Steinle T, Wehler T, Dregely D, Weiss T, Belotelov V I, Stritzker B and Giessen H 2013 Nonreciprocal plasmonics enables giant enhancement of thin-film Faraday rotation *Nat. Commun.* **4** 1599
- [37] Christ A, Tikhodeev S G, Gippius N A, Kuhl J and Giessen H 2003 Waveguide-plasmon polaritons: strong coupling of photonic and electronic resonances in a metallic photonic crystal slab *Phys. Rev. Lett.* **91** 183901
- [38] Christ A, Zentgraf T, Kuhl J, Tikhodeev S G, Gippius N A and Giessen H 2004 Optical properties of planar metallic photonic crystal structures: experiment and theory *Phys. Rev. B* **70** 125113
- [39] Linden S, Kuhl J and Giessen H 2001 Controlling the interaction between light and gold nanoparticles: selective suppression of extinction *Phys. Rev. Lett.* **86** 4688–91
- [40] Floess D, Chin J Y, Kawatani A, Dregely D, Habermeier H-U, Weiss T and Giessen H 2015 Tunable and switchable polarization rotation with non-reciprocal plasmonic thin films at designated wavelengths *Light: Sci. Appl.* **4** e284
- [41] Floess D, Weiss T, Tikhodeev S G and Giessen H 2016 Lorentz nonreciprocal model for hybrid magnetoplasmonics *Phys. Rev. Lett.* **117** 063901
- [42] Floess D, Hentschel M, Weiss T, Habermeier H-U, Jiao J, Tikhodeev S G and Giessen H 2017 Plasmonic analog of electromagnetically induced absorption leads to giant thin film faraday rotation of 14 deg *Phys. Rev. X* **7** 021048
- [43] Floess D 2017 Hybrid plasmonic structures for giant Faraday rotation *PhD Thesis* University of Stuttgart (<https://doi.org/10.18419/opus-9277>)
- [44] Zvezdin A K and Kotov V A 1997 *Modern Magneto-optics and Magneto-optical Materials* (Bristol: IOP Publishing)
- [45] Fujiwara H 2007 *Spectroscopic Ellipsometry: Principles and Applications* (West Sussex: Wiley)
- [46] Jackson J D 1999 *Classical Electrodynamics* 3rd edn (New York: Wiley)
- [47] Yariv A and Yeh P 2007 *Photonics* 6th edn (Oxford: Oxford University Press)
- [48] Fan S *et al* 2012 Comment on ‘Nonreciprocal light propagation in a silicon photonic circuit’ *Science* **335** 38
- [49] Potton R J 2004 Reciprocity in optics *Rep. Prog. Phys.* **67** 717–54
- [50] Fowles G R 1975 *Introduction to Modern Optics* 2nd edn (New York: Dover)
- [51] Hopf F A and Stegeman G I 1992 Applied classical electrodynamics *Volume I: Linear Optics* (Malabar, FL: Krieger)
- [52] Fox M 2010 *Optical Properties of Solids* 2nd edn (Oxford: Oxford University Press)

- [53] Schoenes J 1975 Magneto-optik und elektronische Struktur der magnetisch ordnenden Europiumchalkogenide *Z. Phys. B: Condens. Matter* **20** 345–68
- [54] Yin X, Schäferling M, Metzger B and Giessen H 2013 Interpreting chiral nanophotonic spectra: the plasmonic Born-Kuhn model *Nano Lett.* **13** 6238–43
- [55] Dillon J F 1968 Origin and uses of the Faraday rotation in magnetic crystals *J. Appl. Phys.* **39** 922
- [56] Devreese J T 1980 *Theoretical Aspects and New Developments in Magneto-Optics* (New York: Plenum)
- [57] Schoenes J and Wachter P 1974 Exchange optics in Gd-doped EuO *Phys. Rev. B* **9** 3097–105
- [58] Wachter P 1979 Europium chalcogenides: EuO, EuS, EuSe and EuTe *Handbook on the Physics and Chemistry of Rare Earths* vol 2 (Amsterdam: North-Holland) pp 507–74
- [59] Shen Y R and Bloembergen N 1964 Faraday rotation of rare-earth ions in Ca F<sub>2</sub>. II. Experiments *Phys. Rev.* **257** 5–10
- [60] Wehlius T, Körner T, Leitenmeier S, Heinrich A and Stritzker B 2011 Magneto-optical garnets for integrated optoelectronic devices *Phys. Status Solidi A* **208** 252–63
- [61] Cooper R W, Crossley W A, Page J L and Pearson R F 1968 Faraday rotation in YIG and TbIG *J. Appl. Phys.* **39** 565–7
- [62] Belotelov V I, Akimov I A, Pohl M, Kotov V A, Kasture S, Vengurlekar A S, Gopal A V, Yakovlev D R, Zvezdin A K and Bayer M 2011 Enhanced magneto-optical effects in magnetoplasmonic crystals *Nat. Nanotechnol.* **6** 370–6
- [63] Belotelov V I, Doskolovich L L and Zvezdin A K 2007 Extraordinary magneto-optical effects and transmission through metal-dielectric plasmonic systems *Phys. Rev. Lett.* **98** 077401
- [64] Albrecht G, Kaiser S, Giessen H and Hentschel M 2017 Refractory plasmonics without refractory materials *Nano Lett.* **17** 6402–8
- [65] Albrecht G, Ubl M, Kaiser S, Giessen H and Hentschel M 2018 Comprehensive study of plasmonic materials in the visible and near-infrared: linear, refractory, and nonlinear optical properties *ACS Photonics* **5** 1058
- [66] Shuvaev A M, Astakhov G V, Pimenov A, Brüne C, Buhmann H and Molenkamp L W 2011 Giant magneto-optical Faraday effect in HgTe thin films in the terahertz spectral range *Phys. Rev. Lett.* **106** 107404
- [67] Crassee I, Levallois J, Walter A L, Ostler M, Bostwick A, Rotenberg E, Seyller T, van der Marel D and Kuzmenko A B 2010 Giant Faraday rotation in single- and multilayer graphene *Nat. Phys.* **7** 48–51
- [68] Maier S 2007 *Plasmonics: Fundamentals and Applications* (New York: Springer)
- [69] Tittl A, Giessen H and Liu N 2015 Plasmonic gas and chemical sensing *Nanomater. Nanoarchit.: Complex Rev. Curr. Hot Topics Appl.* **3** 239–72
- [70] Mie G 1908 Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen *Ann. Phys.* **330** 377–445
- [71] Albaladejo S, Gómez-Medina R, Froufe-Pérez L S, Marinchio H, Carminati R, Torrado J F, Armelles G, García-Martín A and Sáenz J J 2010 Radiative corrections to the polarizability tensor of an electrically small anisotropic dielectric particle *Opt. Express* **18** 3556
- [72] Tikhodeev S G, Yablonskii A L, Muljarov E A, Gippius N A and Ishihara T 2002 Quasiguidded modes and optical properties of photonic crystal slabs *Phys. Rev. B* **66** 045102
- [73] Linden S 2001 Kontrolle der Wechselwirkung zwischen Licht und Partikelplasmonen durch selektive Unterdrückung der Extinktion *PhD Thesis* Philipps-Universität Marburg
- [74] Christ A 2005 Optical properties of metallic photonic crystal structures *PhD Thesis* University of Stuttgart
- [75] Tien P K, Ulrich R and Martin R J 1969 Modes of propagating light waves in thin deposited semiconductor films *Appl. Phys. Lett.* **14** 291–4
- [76] Midwinter J E 1970 Evanescent field coupling into a thin-film waveguide *IEEE J. Quantum Electron.* **6** 583–90
- [77] Kittel C 2004 *Introduction to Solid State Physics* (New York: Wiley)
- [78] Armelles G, Cebollada A, García-Martín A, García-Martín J M, González M U, González-Díaz J B, Ferreira-Vila E and Torrado J F 2009 Magnetoplasmonic nanostructures: systems supporting both plasmonic and magnetic properties *J. Opt. A: Pure Appl. Opt.* **11** 114023
- [79] Zheludev N I 2010 The road ahead for metamaterials *Science* **328** 582–3
- [80] Temnov V V 2012 Ultrafast acousto-magneto-plasmonics *Nat. Photon.* **6** 728
- [81] Armelles G, Cebollada A, García-Martín A and González M U 2013 Magnetoplasmonics: magnetoplasmonics: combining magnetic and plasmonic functionalities *Adv. Opt. Mater.* **1** 2
- [82] Stadler B J H and Mizumoto T 2014 Integrated magneto-optical materials and isolators: a review *IEEE Photonics J.* **6** 0600215
- [83] Shoji Y and Mizumoto T 2014 Magneto-optical non-reciprocal devices in silicon photonics *Sci. Technol. Adv. Mater.* **15** 014602
- [84] Maksymov I S 2015 Magneto-plasmonics and resonant interaction of light with dynamic magnetisation in metallic and all-magneto-dielectric nanostructures *Nanomaterials* **5** 577–613
- [85] Maksymov I S 2016 Magneto-plasmonic nanoantennas: basics and applications *Rev. Phys.* **1** 36–51
- [86] Shadrivov I V, Fedotov V A, Powell D A, Kivshar Y S and Zheludev N I 2011 Electromagnetic wave analogue of an electronic diode *New J. Phys.* **13** 033025
- [87] Fan L, Varghese L T, Wang J, Xuan Y, Weiner A M and Qi M 2013 Silicon optical diode with 40 dB nonreciprocal transmission *Opt. Lett.* **38** 1259
- [88] Zhang Y, Li D, Zeng C, Huang Z, Wang Y, Huang Q, Wu Y, Yu J and Xia J 2014 Silicon optical diode based on cascaded photonic crystal cavities *Opt. Lett.* **39** 1370
- [89] Sahoo P K and Joseph J 2013 Optical diode using nonlinear polystyrene ring resonators in two-dimensional photonic crystal structure *Appl. Opt.* **52** 8252–7
- [90] Kong X K, Liu S B, Zhang H F, Dai Y and Yang H 2013 A theoretical study of a compact and highly efficient isolator consisting of nonlinear plasma and matching metamaterials *Laser Phys.* **23** 055404
- [91] Li N and Ren J 2014 Non-reciprocal geometric wave diode by engineering asymmetric shapes of nonlinear materials *Sci. Rep.* **4** 1–6
- [92] Mahmoud A M, Davoyan A R and Engheta N 2015 All-passive nonreciprocal metastructure *Nat. Commun.* **6** 8359
- [93] Miroshnichenko A E, Brasselet E and Kivshar Y S 2010 Reversible optical nonreciprocity in periodic structures with liquid crystals *Appl. Phys. Lett.* **063302** 063302
- [94] Bender N, Factor S, Bodyfelt J D, Ramezani H, Christodoulides D N, Ellis F M and Kottos T 2013 Observation of asymmetric transport in structures with active nonlinearities *Phys. Rev. Lett.* **110** 1–5
- [95] Nazari F, Bender N, Ramezani H, Moravvej-Farshi M K, Christodoulides D N and Kottos T 2014 Optical isolation via PT-symmetric nonlinear Fano resonances *Opt. Express* **22** 9574
- [96] Chang L, Jiang X, Hua S, Yang C, Wen J, Jiang L, Li G, Wang G and Xiao M 2014 Parity-time symmetry and variable optical isolation in active-passive-coupled microresonators *Nat. Photon.* **8** 524–9
- [97] Peng B, Özdemir S K, Lei F, Monifi F, Gianfreda M, Long G L, Fan S, Nori F, Bender C M and Yang L 2014 Parity-time-symmetric whispering-gallery microcavities *Nat. Phys.* **10** 394–8

- [98] Souнас D L and Alù A 2014 Angular-momentum-biased nanorings to realize magnetic-free integrated optical isolation *ACS Photonics* **1** 198–204
- [99] Yu Z and Fan S 2009 Complete optical isolation created by indirect interband photonic transitions *Nat. Photon.* **3** 91–4
- [100] Kamal A, Clarke J and Devoret M H 2011 Noiseless non-reciprocity in a parametric active device *Nat. Phys.* **7** 311–5
- [101] Doerr C R, Dupuis N and Zhang L 2011 Optical isolator using two tandem phase modulators *Opt. Lett.* **36** 4293
- [102] Lira H, Yu Z, Fan S and Lipson M 2012 Electrically driven nonreciprocity induced by interband photonic transition on a silicon chip *Phys. Rev. Lett.* **109** 033901
- [103] Wang D W, Zhou H T, Guo M J, Zhang J X, Evers J and Zhu S Y 2013 Optical diode made from a moving photonic crystal *Phys. Rev. Lett.* **110** 093901
- [104] Doerr C R, Chen L and Vermeulen D 2014 Silicon photonics broadband modulation-based isolator *Opt. Express* **22** 4493
- [105] Tzuan L D, Fang K, Nussenzeig P, Fan S and Lipson M 2014 Non-reciprocal phase shift induced by an effective magnetic flux for light *Nat. Photon.* **8** 701–5
- [106] Kang M S, Butsch A and Russell P St J 2011 Reconfigurable light-driven opto-acoustic isolators in photonic crystal fibre *Nat. Photon.* **5** 549–53
- [107] Shen Z, Zhang Y L, Chen Y, Zou C L, Xiao Y F, Zou X B, Sun F W, Guo G C and Dong C H 2016 Experimental realization of optomechanically induced non-reciprocity *Nat. Photon.* **10** 657–61
- [108] Ruesink F, Miri M A, Alù A and Verhagen E 2016 Nonreciprocity and magnetic-free isolation based on optomechanical interactions *Nat. Commun.* **7** 1–8
- [109] Reiskarimian N and Krishnaswamy H 2016 Magnetic-free non-reciprocity based on staggered commutation *Nat. Commun.* **7** 1–10
- [110] Poulton C G, Pant R, Byrnes A, Fan S, Steel M J and Eggleton B J 2012 Design for broadband on-chip isolator using stimulated Brillouin scattering in dispersion-engineered chalcogenide waveguides *Opt. Express* **20** 21235
- [111] Souнас D, Caloz C and Alù A 2013 Giant non-reciprocity at the subwavelength scale using angular momentum-biased metamaterials *Nat. Commun.* **4** 2407
- [112] Khanikaev A B, Mousavi S H, Shvets G and Kivshar Y S 2010 One-way extraordinary optical transmission and nonreciprocal spoof plasmons *Phys. Rev. Lett.* **105** 126804
- [113] Shoji Y, Mizumoto T, Yokoi H, Hsieh I W and Osgood R M 2008 Magneto-optical isolator with silicon waveguides fabricated by direct bonding *Appl. Phys. Lett.* **92** 071117
- [114] Tien M-C, Mizumoto T, Pintus P, Kromer H and Bowers J E 2011 Silicon ring isolators with bonded nonreciprocal magneto-optic garnets *Opt. Express* **19** 11740
- [115] Ghosh S, Keyvavinia S, Van Roy W, Mizumoto T, Roelkens G and Baets R 2012 Ce:YIG/silicon-on-insulator waveguide optical isolator realized by adhesive bonding *Opt. Express* **20** 1839
- [116] Davoyan A R and Engheta N 2013 Nanoscale plasmonic circulator *New J. Phys.* **15** 083054
- [117] Davoyan A R and Engheta N 2013 Nonreciprocal rotating power flow within plasmonic nanostructures *Phys. Rev. Lett.* **111** 047401
- [118] Scheucher M, Hilico A, Will E, Volz J and Rauschenbeutel A 2016 Quantum optical circulator controlled by a single chirally coupled atom *Science* **354** 1577–80
- [119] Lodahl P, Mahmoodian S, Stobbe S, Rauschenbeutel A, Schneeweiss P and Volz J 2017 Chiral quantum optics *Nature* **541** 473–80
- [120] Gong S H, Alpegiani F, Sciacca B, Garnett E C and Kuipers L 2018 Nanoscale chiral valley-photon interface through optical spin-orbit coupling *Science* **359** 443–7
- [121] Chervy T, Azzini S, Lorchat E, Wang S, Gorodetski Y, Hutchison J A, Berciaud S, Ebbesen T W and Genet C 2018 Chiral coupling of valley excitons in a transition metal dichalcogenide monolayer *ACS Photonics* **5** 1281–7
- [122] Christofi A, Kawaguchi Y, Alù A and Khanikaev A B 2018 Giant enhancement of Faraday rotation due to electromagnetically induced transparency in all-dielectric magneto-optical metasurfaces *Opt. Lett.* **43** 1838
- [123] Kuznetsov A I, Miroshnichenko A E, Brongersma M L, Kivshar Y S and Luk'yanchuk B 2016 Optically resonant dielectric nanostructures *Science* **354** aag2472
- [124] Alaei R, Filter R, Lehr D, Lederer F and Rockstuhl C 2015 A generalized Kerker condition for highly directive nanoantennas *Opt. Lett.* **40** 2645
- [125] Menzel C, Helgert C, Rockstuhl C, Kley E-B, Tünnermann A, Pertsch T and Lederer F 2010 Asymmetric transmission of linearly polarized light at optical metamaterials *Phys. Rev. Lett.* **104** 1–4
- [126] Fedotov V A, Mladyonov P L, Prosvirnin S L, Rogacheva A V, Chen Y and Zheludev N I 2006 Asymmetric propagation of electromagnetic waves through a planar chiral structure *Phys. Rev. Lett.* **97** 1–4
- [127] Schwanecke A S, Fedotov V A, Khardikov V V, Prosvirnin S L, Chen Y and Zheludev N I 2008 Nanostructured metal film with asymmetric optical transmission *Nano Lett.* **8** 2940–3
- [128] Drezet A, Genet C, Laluet J-Y and Ebbesen T W 2008 Optical chirality without optical activity: how surface plasmons give a twist to light *Opt. Express* **16** 12559
- [129] Zhukovsky S V, Novitsky A V and Galynsky V M 2009 Elliptical dichroism: operating principle of planar chiral metamaterials *Opt. Lett.* **34** 1988–90
- [130] Singh R, Plum E, Menzel C, Rockstuhl C, Azad A K, Cheville R A, Lederer F, Zhang W and Zheludev N I 2009 Terahertz metamaterial with asymmetric transmission *Phys. Rev. B* **80** 153104
- [131] Feng H Y, Luo F, Kekesi R, Granados D, Meneses-Rodríguez D, García J M, García-Martín A, Armelles G and Cebollada A 2014 Magnetoplasmonic nanorings as novel architectures with tunable magneto-optical activity in wide wavelength ranges *Adv. Opt. Mater.* **2** 612–7
- [132] Maccaferri N *et al* 2013 Polarizability and magnetoplasmonic properties of magnetic general nanoellipsoids *Opt. Express* **21** 9875
- [133] Lacoste D and van Tiggelen B A 1999 Optics of a Faraday-active Mie sphere *J. Opt. Soc. Am. A* **15** 17
- [134] Tarento R J, Bennemann K H, Joyes P and Van de Walle J 2004 Mie scattering of magnetic spheres *Phys. Rev. E* **69** 1–5
- [135] Berger A, Alcaraz De La Osa R, Suszka A K, Pancaldi M, Saiz J M, Moreno F, Oopen H P and Vavassori P 2015 Enhanced magneto-optical edge excitation in nanoscale magnetic disks *Phys. Rev. Lett.* **115** 1–6
- [136] Hui P M and Stroud D 1987 Theory of Faraday-rotation by dilute suspensions of small particles *Appl. Phys. Lett.* **50** 950–2
- [137] Kravets V G, Petford-long A K and Kravetz A F 1999 Study of optical and magneto-optical properties of CoFe-HfO<sub>2</sub> granular magnetic films *Physica E* **4** 292–9
- [138] Melle S, Menéndez J L, Armelles G, Navas D, Vázquez M, Nielsch K, Wehrspohn R B and Gösele U 2003 Magneto-optical properties of nickel nanowire arrays *Appl. Phys. Lett.* **83** 4547–9
- [139] Du G X, Mori T, Suzuki M, Saito S, Fukuda H and Takahashi M 2010 Evidence of localized surface plasmon

- enhanced magneto-optical effect in nanodisk array *Appl. Phys. Lett.* **96** 2008–11
- [140] Chen J *et al* 2011 Plasmonic nickel nanoantennas *Small* **7** 2341–7
- [141] Maccaferri N *et al* 2013 Tuning the magneto-optical response of nanosize ferromagnetic Ni disks using the phase of localized plasmons *Phys. Rev. Lett.* **111** 1–5
- [142] Bonanni V, Bonetti S, Pakizeh T, Pirzadeh Z, Chen J, Nogués J, Vavassori P, Hillenbrand R, Åkerman J and Dmitriev A 2011 Designer magnetoplasmonics with nickel nanoferrimagnets *Nano Lett.* **11** 5333–8
- [143] Smith D A and Stokes K L 2006 Discrete dipole approximation for magneto-optical scattering calculations *Opt. Express* **14** 5746–54
- [144] Pineider F, Campo G, Bonanni V, Fernández C, Mattei G, Caneschi A, Gatteschi D and Sangregorio C 2013 Circular magnetoplasmonic modes in gold nanoparticles *Nano Lett.* **13** 4785–9
- [145] Maccaferri N, Gregorczyk K E de Oliveira T, Kataja M, van Dijken S, Pirzadeh Z, Dmitriev A, Åkerman J, Knez M and Vavassori P 2015 Ultrasensitive and label-free molecular-level detection enabled by light phase control in magnetoplasmonic nanoantennas *Nat. Commun.* **6** 6150
- [146] Li Y, Zhang Q, Nurmikko A V and Sun S 2005 Enhanced magneto-optical response in dumbbell-like Ag-CoFe 20 4 nanoparticle pairs *Nano Lett.* **5** 1689–92
- [147] González-Díaz J B, García-Martín A, García-Martín J M, Cebollada A, Armelies G, Sepúlveda B, Alaverdyan Y and Käll M 2008 Plasmonic Au/Co/Au nanosandwiches with enhanced magneto-optical activity *Small* **4** 202–5
- [148] Armelies G *et al* 2008 Localized surface plasmon resonance effects on the magneto-optical activity of continuous Au/Co/Au trilayers *Opt. Express* **16** 16104–12
- [149] Torrado J F, González-Díaz J B, González M U, García-Martín A and Armelies G 2010 Magneto-optical effects in interacting localized and propagating surface plasmon modes *Opt. Express* **18** 15635–42
- [150] Armelies G, Cebollada A, Feng H Y, García-Martín A, Meneses-Rodríguez D, Zhao J and Giessen H 2015 Interaction effects between magnetic and chiral building blocks: a new route for tunable magneto-chiral plasmonic structures *ACS Photonics* **2** 1272–7
- [151] Almpanis E, Pantazopoulos P-A, Papanikolaou N, Yannopapas V and Stefanou N 2016 Metal-nanoparticle arrays on a magnetic garnet film for tunable plasmon-enhanced Faraday rotation *J. Opt. Soc. Am. B* **33** 2609–16
- [152] Uchida H, Masuda Y, Fujikawa R, Baryshev A V and Inoue M 2009 Large enhancement of Faraday rotation by localized surface plasmon resonance in Au nanoparticles embedded in Bi:YIG film *J. Magn. Magn. Mater.* **321** 843–5
- [153] Baryshev A V, Merzlikin A M and Aryshev A V B 2016 Tunable plasmonic thin magneto-optical wave plate *J. Opt. Soc. Am. B* **33** 1399
- [154] Baranov D G, Vinogradov A P and Lisyansky A A 2015 Magneto-optics enhancement with gain-assisted plasmonic subdiffraction chains *J. Opt. Soc. Am. B* **32** 281
- [155] Zubritskaya I, Maccaferri N, Ezeiza X I, Vavassori P and Dmitriev A 2017 Magnetic control of the chiroptical plasmonic surfaces *Nano Lett.* **18** 302
- [156] Eslami S and Gibbs J G 2014 Chiral nanomagnets *ACS Photonics* **1** 1231–6
- [157] Armelies G, Cebollada A, García-Martín A, González M U, García F, Meneses-Rodríguez D, de Sousa N and Froufe-Pérez L S 2013 Mimicking electromagnetically induced transparency in the magneto-optical activity of magnetoplasmonic nanoresonators *Opt. Express* **21** 27356
- [158] Dahlin A B, Tegenfeldt J O and Höök F 2006 Improving the instrumental resolution of sensors based on localized surface plasmon resonance *Anal. Chem.* **78** 4416–23
- [159] Temnov V, Armelies G, Woggon U, Guzатов D, Cebollada A, García-Martín A, Thomay T, Leitenstorfer A and Bratschkitsch R 2010 Active magneto-plasmonics in hybrid metal-ferromagnet structures *Nat. Photon.* **4** 107–11
- [160] Ebbesen T W, Lezec H J, Ghaemi H F, Thino T and Wolff P A 1998 Extraordinary optical transmission through sub-wavelength hole arrays *Nature* **391** 69–71
- [161] Atwater H 2007 The promise of plasmonics *Sci. Am.* **296** 56–62
- [162] Fan S 2010 Nanophotonics: magnet-controlled plasmons *Nat. Photon.* **4** 76–7
- [163] Martín-Becerra D *et al* 2010 Enhancement of the magnetic modulation of surface plasmon polaritons in Au/Co/Au films *Appl. Phys. Lett.* **97** 183114
- [164] Ferguson P E, Stafsudd O M and Wallis R F 1977 Enhancement of the transverse Kerr magneto-optic effect by surface magnetoplasma waves *Physica B+C* **89** 91–4
- [165] Safarov V I, Kosobukin V A, Hermann C, Lampel G, Peretti J and Marlière C 1994 Magneto-optical effects enhanced by surface plasmons in metallic multilayer films *Phys. Rev. Lett.* **73** 3584–7
- [166] Kochergin V E, Toporov A Yu and Valeiko M V 1998 Polariton enhancement of the Faraday magneto-optic effect *J. Exp. Theor. Phys. Lett.* **68** 400–3
- [167] Ferreiro-Vila E, Sueiro X M B, González-Díaz J B, García-Martín A, García-Martín J M, Navarro A C, Reig G A, Rodríguez D M and Sandoval E M 2008 Surface plasmon resonance effects in the magneto-optical activity of Ag-Co-Ag trilayers *IEEE Trans. Magn.* **44** 3303–6
- [168] Ferreiro-Vila E *et al* 2009 Intertwined magneto-optical and plasmonic effects in Ag/Co/Ag layered structures *Phys. Rev. B* **80** 125132
- [169] Yang K, Clavero C, Skuza J R, Varela M and Lukaszew R A 2010 Surface plasmon resonance and magneto-optical enhancement on Au-Co nanocomposite thin films *J. Appl. Phys.* **107** 103924
- [170] Clavero C, Yang K, Skuza J R and Lukaszew R A 2010 Magnetic field modulation of intense surface plasmon polaritons *Opt. Express* **18** 7743
- [171] Ctistis G, Papaioannou E, Patoka P, Gutek J, Fumagalli P and Giersig M 2009 Optical and magnetic properties of hexagonal arrays of subwavelength holes in optically thin cobalt films *Nano Lett.* **9** 1–6
- [172] González-Díaz J B, García-Martín J M, García-Martín A, Navas D, Asenjo A, Vázquez M, Hernández-Vélez M and Armelies G 2009 Plasmon-enhanced magneto-optical activity in ferromagnetic membranes *Appl. Phys. Lett.* **94** 263101
- [173] Torrado J F, Papaioannou E Th, Ctistis G, Patoka P, Giersig M, Armelies G and García-Martín A 2010 Plasmon induced modification of the transverse magneto-optical response in Fe antidot arrays *Phys. Status Solidi* **4** 271–3
- [174] Papaioannou E T, Kapaklis V, Giersig M, Fumagalli P, García-Martín A, Ferreiro-Vila E and Ctistis G 2010 Magneto-optic enhancement and magnetic properties in Fe antidot films with hexagonal symmetry *Phys. Rev. B* **81** 1–5
- [175] Papaioannou E T, Kapaklis V, Melander E, Hjörvarsson B, Pappas S D, Patoka P, Giersig M, Fumagalli P, García-Martín A and Ctistis G 2011 Surface plasmons and magneto-optic activity in hexagonal Ni anti-dot arrays *Opt. Express* **19** 23867–77
- [176] Gevorkian Z and Gasparian V 2014 Plasmon-enhanced Faraday rotation in thin films *Phys. Rev. A* **89** 023830

- [177] Maccaferri N, Inchausti X, García-Martín A, Cuevas J C, Tripathy D, Adeyeye A O and Vavassori P 2015 Resonant enhancement of magneto-optical activity induced by surface plasmon polariton modes coupling in 2D magnetoplasmonic crystals *ACS Photonics* **2** 1769
- [178] Belotelov V I, Bykov D A, Doskolovich L L, Kalish A N and Zvezdin A K 2009 Extraordinary transmission and giant magneto-optical transverse Kerr effect in plasmonic nanostructured films *J. Opt. Soc. Am. B* **26** 1594
- [179] Belotelov V I, Bykov D A, Doskolovich L L, Kalish A N, Kotov V A and Zvezdin A K 2009 Giant magneto-optical orientational effect in plasmonic heterostructures *Opt. Lett.* **34** 398–400
- [180] Belotelov V *et al* 2013 Plasmon-mediated magneto-optical transparency *Nat. Commun.* **4** 2128
- [181] Akulshin A M, Barreiro S and Lezama A 1998 Electromagnetically induced absorption and transparency due to resonant two-field excitation of quasidegenerate levels in Rb vapor *Phys. Rev. A* **57** 2996–3002
- [182] Lezama A, Barreiro S and Akulshin A M 1999 Electromagnetically induced absorption *Phys. Rev. A* **59** 4732–5
- [183] Suits J C and Argyle B E 1965 Paramagnetic Faraday rotation of EuSe *J. Appl. Phys.* **36** 1251–2
- [184] Schoenes J and Wachter P 1977 High field magneto-optical study of EuSe and EuTe *Physica B* **89B** 155–8
- [185] Suits J C and Argyle B E 1965 Magnetic birefringence of EuSe *Phys. Rev. Lett.* **14** 687–9
- [186] Mahan E 2000 *Physical Vapor Deposition of Thin Films* (New York: Wiley) p 336
- [187] Koshizuka N 1987 Research activities on magneto-optical devices in Japan *IEEE Trans. Magn.* **23** 3473
- [188] Ohido A, Sugawara T, Yamasawa K, Kakei S, Shimakawa K and Hosoya K 2009 Magnetic garnet material, faraday rotator, optical device, bismuth-substituted rare earth-iron-garnet single-crystal film and method for producing the same and crucible for producing the same *US Patent* 7,517,406
- [189] Shirai K and Takeda N 1999 Bismuth-substituted rare earth iron garnet single crystal film *US Patent* 5,925,474
- [190] Shirai K and Takeda N 2002 Faraday rotator for use with high energy lasers *US Patent* 6,411,641
- [191] View C, Carcenac F, Pépin A, Chen Y, Mejias M, Lebib A, Manin-Ferlazzo L, Couraud L and Launois H 2000 Electron beam lithography: resolution limits and applications *Appl. Surf. Sci.* **164** 111–7
- [192] Weiss T, Gippius N A, Tikhodeev S G, Granet G and Giessen H 2011 Derivation of plasmonic resonances in the Fourier modal method with adaptive spatial resolution and matched coordinates *J. Opt. Soc. Am. A* **28** 238–44
- [193] Johnson P B and Christy R W 1972 Optical constants of the noble metals *Phys. Rev. B* **6** 4370
- [194] Güntherodt G, Schoenes J and Wachter P 1970 Optical constants of the Eu chalcogenides above and below the magnetic ordering temperatures *J. Appl. Phys.* **41** 1083
- [195] Ross W E 1983 Two-dimensional magneto-optic spatial light modulator for signal processing *Opt. Eng.* **22** 485–90
- [196] Sun L, Jiang S and Marciante J R 2010 All-fiber optical magnetic-field sensor based on Faraday rotation in highly terbium-doped fiber *Opt. Express* **18** 5407–12
- [197] Deeter M N, Rose A H and Day G W 1990 Fast, sensitive magnetic-field sensors based on the Faraday effect in YIG *J. Lightwave Technol.* **8** 1838–42
- [198] Guo H C, Nau D, Radke A, Zhang X P, Stodolka J, Yang X L, Tikhodeev S G, Gippius N A and Giessen H 2005 Large-area metallic photonic crystal fabrication with interference lithography and dry etching *Appl. Phys. B* **81** 271–5
- [199] Bagheri S, Giessen H and Neubrech F 2014 Large-area antenna-assisted SEIRA substrates by laser interference lithography *Adv. Opt. Mater.* **2** 1050–6
- [200] Wang X, Venugopal G, Zeng J, Chen Y, Lee D H, Litchinitser N M and Cartwright A N 2011 Optical fiber metamagnetics *Opt. Express* **19** 19813–21
- [201] Lin Y, Zou Y, Mo Y, Guo J and Lindquist R G 2010 E-beam patterned gold nanodot arrays on optical fiber tips for localized surface plasmon resonance biochemical sensing *Sensors* **10** 9397–406
- [202] Zeng X, Jradi S, Proust J, Bachelot R, Zhang Z P, Royer P and Plain J 2011 Direct functionalization of an optical fiber by a plasmonic nanosensor *Opt. Lett.* **36** 2919–21
- [203] Dentz D J, Puttbach R C and Belt R F 1974 Terbium gallium garnet for Faraday effect Devices *AIP Conf. Proc.* **18** 954–8
- [204] Ahn K and Suits J 1967 Preparation and properties of EuO films *IEEE Trans. Magn.* **3** 2–4
- [205] Shafer M W 1968 Studies of curie-point increases in EuO *J. Appl. Phys.* **39** 588–90
- [206] Doost M B, Langbein W and Muljarov E A 2014 Resonant-state expansion applied to three-dimensional open optical systems *Phys. Rev. A* **90** 013834
- [207] Weiss T, Mesch M, Schäferling M, Giessen H, Langbein W and Muljarov E A 2016 From dark to bright: first-order perturbation theory with analytical mode normalization for plasmonic nanoantenna arrays applied to refractive index sensing *Phys. Rev. Lett.* **116** 237401
- [208] Weiss T, Granet G, Gippius N A, Tikhodeev S G and Giessen H 2009 Matched coordinates and adaptive spatial resolution in the Fourier modal method *Opt. Express* **17** 8051–61
- [209] Akimov A B, Gippius N A and Tikhodeev S G 2011 Optical fano resonances in photonic crystal slabs near diffraction threshold anomalies *JETP Lett.* **93** 427–30
- [210] Metzger B, Hentschel M, Lippitz M and Giessen H 2012 Third-harmonic spectroscopy and modeling of the nonlinear response of plasmonic nanoantennas *Opt. Lett.* **37** 4741
- [211] Taubert R, Hentschel M, Kästel J and Giessen H 2012 Classical analogue of electromagnetically induced absorption in plasmonics *Nano Lett.* **12** 1367
- [212] Taubert R, Hentschel M and Giessen H 2013 Plasmonic analog of electromagnetically induced absorption: simulations, experiments, and coupled oscillator analysis *J. Opt. Soc. Am. B* **30** 3123
- [213] Harris S E 1997 Electromagnetically induced transparency *Phys. Today* **50** 36
- [214] Boller K J, Imamoglu A and Harris S E 1991 Observation of electromagnetically induced transparency *Phys. Rev. Lett.* **66** 2593–6
- [215] Liu N, Langguth L, Weiss T, Kästel J, Fleischhauer M, Pfau T and Giessen H 2009 Plasmonic analogue of electromagnetically induced transparency at the Drude damping limit *Nat. Mater.* **8** 758–62
- [216] Liu N, Weiss T, Mesch M, Langguth L, Eigenthaler U, Hirscher M, Sönnichsen C and Giessen H 2010 Planar metamaterial analogue of electromagnetically induced transparency for plasmonic sensing *Nano Lett.* **10** 1103–7
- [217] Verellen N, Van Dorpe P, Huang C, Lodewijks K, Vandenbosch G A E, Lagae L and Moshchalkov Victor V 2011 Plasmon line shaping using nanocrosses for high sensitivity localized surface plasmon resonance sensing *Nano Lett.* **11** 391–7
- [218] Lassiter J B, Sobhani H, Fan J A, Kundu J, Capasso F, Nordlander P and Halas N J 2010 Fano resonances in plasmonic nanoclusters: geometrical and chemical tunability *Nano Lett.* **10** 3184–9

- [219] Yanik M F, Suh W, Wang Z and Fan S 2004 Stopping light in a waveguide with an all-optical analog of electromagnetically induced transparency *Phys. Rev. Lett.* **93** 233903
- [220] Totsuka K, Kobayashi N and Tomita M 2007 Slow Light in Coupled-Resonator-Induced Transparency *Phys. Rev. Lett.* **98** 213904
- [221] Schönhardt A, Nau D, Bauer C, Christ A, Gräbeldinger H and Giessen H 2017 Phase-resolved pulse propagation through metallic photonic crystal slabs: plasmonic slow light *Phil. Trans. A* **275** 0065
- [222] Maleki L, Matsko A B, Savchenkov A A and Ilchenko V S 2004 Tunable delay line with interacting whispering-gallery-mode resonators *Opt. Lett.* **29** 626–8
- [223] Papanikolaou N, Fedotov V A, Zheludev N I and Prosvirnin S L 2008 Metamaterial analog of electromagnetically induced transparency *Phys. Rev. Lett.* **101** 253903
- [224] Fleischhauer M and Marangos J P 2005 Electromagnetically induced transparency: Optics in coherent media *Rev. Mod. Phys.* **77** 633–73
- [225] Metzger B, Schumacher T, Hentschel M, Lippitz M and Giessen H 2014 Third harmonic mechanism in complex plasmonic fano structures *ACS Photonics* **1** 471–6
- [226] Tanaka K, Tatehata N, Fujita K and Hirao K 2001 Preparation and Faraday effect of EuS microcrystal-embedded oxide thin films *J. Appl. Phys.* **89** 2213
- [227] Reddy H, Guler U, Kildishev A V, Boltasseva A and Shalaev V M 2016 Temperature-dependent optical properties of gold thin films *Opt. Mater. Express* **6** 2776
- [228] Leviton D B and Frey B J 2006 Temperature-dependent absolute refractive index measurements of synthetic fused silica *Proc. SPIE* **6273** 62732K
- [229] Hayashi H, Iwasa S, Vasa N J and Yoshitake T 2002 Fabrication of Bi-doped YIG optical thin for electric current sensor by pulsed laser deposition *Appl. Surf. Sci.* **198** 463–6
- [230] Zentgraf T, Zhang S, Oulton R F and Zhang X 2009 Ultranarrow coupling-induced transparency bands in hybrid plasmonic systems *Phys. Rev. B* **80** 195415
- [231] Li L 2003 Fourier modal method for crossed anisotropic gratings with arbitrary permittivity and permeability tensors *J. Opt. A: Pure Appl. Opt.* **5** 345
- [232] Sagawa M, Fujimura S, Togawa N, Yamamoto H and Matsuura Y 1984 New material for permanent magnets on a base of Nd and Fe *J. Appl. Phys.* **55** 2083–7
- [233] Castaldi G, Galdi V, Alù A and Engheta N 2012 Nonlocal transformation optics *Phys. Rev. Lett.* **108** 063902



**Harald Giessen** (\*1966) graduated from Kaiserslautern University with a diploma in Physics and obtained his M.S. and Ph.D. in Optical Sciences from the University of Arizona in 1995 as J.W. Fulbright scholar. After a postdoc at the Max-Planck-Institute for Solid State Research in Stuttgart he moved to Marburg as assistant professor. From 2001–2004, he was associate professor at the University of Bonn. Since 2005, he is full professor and holds the Chair for Ultrafast Nanooptics in the Department of Physics at the University of Stuttgart. He is also co-chair of the Stuttgart Center of Photonics Engineering, SCoPE. He was guest researcher at the University of Cambridge, and guest professor at the University of Innsbruck and the University of Sydney, at A\*Star, Singapore, as well as at Beijing University of Technology. He is associated researcher at the Center for Disruptive

Photonic Technologies at Nanyang Technical University, Singapore. He received an ERC Advanced Grant in 2012 for his work on complex nanoplasmonics. He was co-chair (2014) and chair (2016) of the Gordon Conference on Plasmonics and Nanophotonics. He was general chair of the conference Photonics Europe (Strasbourg 2018) and is co-chair of the biannual conference NanoMeta in Seefeld, Austria. He is on the advisory board of the journals “Advanced Optical Materials”, “Nanophotonics: The Journal”, “ACS Photonics”, “ACS Sensors”, and “Advanced Photonics”. He is a topical editor for ultrafast nanooptics, plasmonics, and ultrafast lasers and pulse generation of the journal “Light: Science & Applications” of Nature Publishing Group. He is a Fellow of the Optical Society of America. His research interests include Ultrafast Nano-Optics, Plasmonics, Metamaterials, 3D Printed Micro- and Nano-Optics, Novel mid-IR Ultrafast Laser Sources, Applications in Microscopy, Biology, and Sensing.



**Dominik Floess** (\*1986) graduated from the Karlsruhe Institute of Technology (KIT) with a German Diploma in Physics and obtained the degree Master of Physics (MPhys) from the Heriot-Watt University in Edinburgh, United Kingdom. In his early research projects he worked on optical whispering gallery resonators for biosensing and the development of quantum algorithms. He received his Ph.D. in Physics from the University of Stuttgart. In his dissertation he investigated nonreciprocity in plasmonic nanostructures for enhanced Faraday rotation in sub-wavelength structures.