The resonant-state expansion, a recently developed powerful method in electrodynamics, is generalized here for open optical systems containing magnetic, chiral, or bi-anisotropic materials. It is shown that the key matrix eigenvalue equation of the method remains the same, but the matrix elements of the perturbation now contain variations of the permittivity, permeability, and bi-anisotropy tensors. A general normalization of resonant states in terms of the electric and magnetic fields is presented.

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The resonant state expansion (RSE) is a novel powerful theoretical method that has been recently developed in electrodynamics [1]. The RSE is a rigorous perturbation theory that is not limited to small perturbations and warrants an efficient calculation of all resonant states (RSs) of an open optical system in an arbitrarily selected spectral range. This calculation is based on knowing the RSs of another, so-called basis system, which is usually (but not necessarily) simpler than the system of interest, ideally having an exact analytic solution. The RSE was verified and tested on optical systems of different shape and dimensionality [2–5], demonstrating its superior computational efficiency [3,5] compared to available numerical methods, such as finite-difference time-domain [6,7], finite element [8], and the aperiodic Fourier modal method [9,10].

Being originally introduced in nuclear physics almost a century ago [11,12], RSs in electrodynamics present eigen-solutions of Maxwell's equations satisfying outgoing boundary conditions, which correspond to electromagnetic excitations decaying in time, with the electromagnetic energy leaking out of the system. This leakage, however, causes an exponential growth of the RS wave function with distance, so that the standard normalization used, for example, for bound states in quantum mechanics or for waveguide modes in optics, diverges. While the correct normalization for scalar fields was known [13], expressions for the normalization of the electromagnetic vector fields of the RSs, intensively used in the literature [14,15], are only approximate, as has been recently clarified [16,17]. The correct normalization of RSs in finite optical systems, which is a cornerstone of the RSE, was presented in the very first paper on the method [1] and was later generalized to arbitrary systems with frequency dispersion of the permittivity [18]. Recently, it has been used to formulate an exact theory of the Purcell effect [16]. Furthermore, the exact normalization was extended to photonic crystal structures [19,20] and applied to resonantly enhanced refractive index sensing using the RSE with only one and two RSs in the basis.

The RSE has also been generalized to optical systems with frequency dispersion of the permittivity [18] without affecting the computational complexity, which is a very important step towards describing realistic materials and specifically plasmonic effects. This was achieved by treating the dispersion as an analytical function with a finite number of simple poles in the lower half-plane of the complex frequency, known in the literature as the generalized Drude–Lorentz model [21].

So far, the RSE was applied to non-magnetic optical systems ($\mu = 1$), which are fully described by the wave equation containing solely the electric field, the permittivity tensor, and an electric current. Naturally, the RS normalization and the RSE itself dealing with perturbations of the permittivity were formulated in terms of the electric field only. However, the most general materials with local responses are bi-anisotropic and have non-zero magnetic susceptibility and coupling tensors between the electric and magnetic fields, including the chiral optical activity and circular dichroism [22]. Describing such systems, which include but are not limited to metamaterials [23], chiral plasmonics [24,25], and chiral sensors, is of growing interest. Therefore, it is crucial to have a general formulation of the RSE and the RS normalization, in which the electric and magnetic fields...