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Abstract

The Faraday effect describes the phenomenon that a magnetized material can alter the polarization state of transmitted light. Interestingly, unlike most light-matter interactions in nature, it breaks Lorentz reciprocity. This exceptional behavior is utilized for applications such as optical isolators, which are core elements in communication and laser systems. While there is high demand for sub-micron nonreciprocal photonic devices, the realization of such systems is extremely challenging as conventional magneto-optic materials only provide weak magneto-optic response within small volumes.

Plasmonics could be a key to overcome this hurdle in the future: over the last years there have been several lines of work demonstrating that different types of metallic nanostructures can be utilized to greatly enhance the magneto-optic response of conventional materials. In this review we give an overview over the state of the art in the field and highlight recent developments on hybrid plasmonic Faraday rotators. Our discussions are mainly focused on the visible and near-infrared wavelength regions and cover both experimental realizations as well as analytical descriptions. Special attention will be paid to recent developments on hybrid plasmonic thin film systems consisting of gold and europium chalcogenides.

Keywords: plasmonics, magneto-optics, Faraday effect, nonreciprocity, nanophotonics

(Some figures may appear in colour only in the online journal)
The presence of a static magnetic field influences the optical properties of certain materials that are referred to as magneto-optic (MO) materials. This gives rise to several MO effects, which distinguishes it from effects such as optical activity [6]. MO effects are actually the only practical way to break Lorentz reciprocity in passive optical systems, since other approaches rely on either nonlinear effects [7] or time modulation [8]. For that reason, Faraday rotators are widely utilized as core elements in nonreciprocal optical devices such as optical isolators [9–11], which require a Faraday rotation of 45°. As such, Faraday rotators are essential components in a multitude of optical systems, including optical telecommunication networks [12–15] and laser systems [16–18].

Most systems that involve Faraday rotators have recently undergone massive miniaturization. Hence, there is a high demand for down-sized Faraday rotators [19, 20]. Especially in laser systems, optical isolators are often the limiting components when it comes to device miniaturization [19]. However, the miniaturization of optical isolators is extremely challenging, since, according to equation (1), a minimum optical path length must be acquired to reach the required 45° Faraday rotation. A promising approach to overcome this problem is to combine MO materials with plasmonic nanostructures. Over the last years there have been several lines of work demonstrating that plasmonic nanostructures can be utilized to enhance the magneto-optic response of conventional materials significantly.

The first approaches to enhance Faraday rotation were based on MO nanoparticles [21], photonic crystals [22–26], and microcavities [27–29] but were hampered by either weak MO effects or relatively large structure sizes that are also difficult to fabricate. There have also been efforts to enhance the rather weak MO Kerr effect by means of ferromagnetic metallic nanoparticles [30–33], photonic crystals [34] and subwavelength dielectric MO gratings [35]. Considerable attention was received by the work by Chin et al in 2013, where the Faraday rotation of a dielectric bismuth-iron-garnet (BIG) thin film is enhanced by attaching a metal grating [36], resulting in Faraday rotation of about 0.8° for a 215 nm thick structure. Here, the strong Faraday rotation originates from the interaction between waveguide resonances in the BIG thin film and localized surface plasmon resonances (LSPRs) in the metal grating [37–39]. At that time, this performance was already a remarkable achievement considering that conventional Faraday rotators require centimeter sized crystals to achieve 45° rotation. However, in the following years there has been a series of works resulting in even thinner Faraday rotators with one order of magnitude larger rotation and more functionality [40–42]. As the performance of these nanoscopic structures is not far away from meeting the requirements of a Faraday isolator, the concept of hybrid magnetoplasmonic crystals could become a key ingredient to realize nonreciprocal photonic devices in a highly integrated environment.

In this review we focus on recent developments on magnetoplasmonic crystal Faraday rotators and put them into the context of nonreciprocal plasmonics and other magneto-optic nanostructures. Our discussion will cover both experimental realizations as well as analytical descriptions. We start with a theoretical primer in section 2, where we introduce the basic physical concepts involved in hybrid magnetoplasmonic systems. This includes magneto-optic effects, plasmonics, and the optical properties of corrugated waveguides. After that, in section 3 we give an overview over the most relevant works in the field of nonreciprocal nanophotonics and magnetoplasmonics. In the sections 4–6 we pay special attention to the most recent experimental and theoretical advances within the realm of hybrid magnetoplasmonics for giant Faraday rotation. Major parts of this review have been adapted with permission from the PhD thesis by Dominik Floess [43] and from the articles [40–42].
2. Fundamentals

This section provides a basic discussion of the physical phenomena such as magneto-optic effects, plasmonics, and the modal dispersion of grating-waveguide structures. The concepts introduced here will be the basis for the discussions in the following sections. This section has been adapted with permission form [43].

2.1. Polarization of light

First, the mathematical foundation and nomenclature for describing the polarization state of classical electromagnetic waves is introduced. The relations compiled in this section are the basis for the discussions throughout this review. The derivations and naming conventions in this section are adapted from Zvezdin and Kotov [44].

A plane electromagnetic wave propagating in z-direction can be written in the form

$$\mathbf{E}(z, t) = \begin{bmatrix} a \cos(kz - \omega t) \\ b \cos(kz - \omega t + \delta) \end{bmatrix},$$

where $\omega$ is the angular frequency and $k$ the wave number [44]. As an alternative formulation to (2) it is often more convenient to use a notation which is based on the assumption that the actual physical electric field is given by the real part of the complex electric field

$$\mathbf{E}(z, t) = \hat{\mathbf{E}} e^{i(kz - \omega t)}$$

with

$$\hat{\mathbf{E}} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix}.$$  \hfill (3)

As in the definition of the real electric field (2), the amplitudes $a$ and $b$, as well as the phase angle $\delta$ are assumed to be real. It is immediately clear that the real part of $\mathbf{E}(z, t)$ in (3) is equal to $\mathbf{E}(z, t)$ in (2).

The electromagnetic wave described by the general expressions (3) and (4) can be interpreted as the superposition of two waves, where one is oscillating only in $x$ direction and the other in $y$ direction. However, depending on the phase $\delta$ between the two partial waves, the resulting field vector (3) performs an elliptical motion. This is illustrated by figure 1. It shows a parametric plot of the real part of the electric field, where the varied parameter is the propagator term $\omega t - kz$. The resulting shape is an ellipse, which is fully characterized by the two angles $\theta$ and $\psi$. While $\theta$ determines the tilt of the polarization ellipse with respect to the $x$-axis, the angle $\psi$ describes the degree of ellipticity. If $\psi = 0$, the wave is linearly polarized, whereas for $|\psi| = \pi$ the wave is circularly polarized. Furthermore, the handedness of the circular polarization can be expressed via the sign of $\psi$, i.e. $\psi = (+)\pi$ corresponds to right(left)-handed circular polarization. The two angles are related to the coefficients in (2) in the following way [44]:

$$\tan(2\theta) = \frac{2 \text{Re}(\chi)}{1 - |\chi|^2}$$ \hfill (5a)

$$\tan(2\psi) = \frac{2 \text{Im}(\chi)}{1 - |\chi|^2}$$ \hfill (5b)

For the important case of $|\chi| \ll 1$ the equations (5) become

$$\theta \approx \text{Re}(\chi)$$ \hfill (7a)

$$\psi \approx \text{Im}(\chi).$$ \hfill (7b)

While $\theta$ and $\psi$ fully determine the polarization state of a plane wave they are not directly accessible in experiment. The reason is that the electric and magnetic fields of a light wave usually cannot be measured directly and have to be derived from time-averaged intensity measurements. For that reason, the Stokes formalism is an important alternative description of polarization states as it is only based on time-averaged intensities. In this formalism the polarization state of light can be expressed by means of the four Stokes parameters [44, 45]

$$S_0 = I_x + I_y$$ \hfill (8a)

$$S_1 = I_x - I_y$$ \hfill (8b)

$$S_2 = I_{45^\circ} + I_{-45^\circ}$$ \hfill (8c)

$$S_3 = I_R + I_L.$$ \hfill (8d)

The quantities $I_x$ and $I_y$ correspond to the time averaged intensity of the $x$- and $y$-polarized components of the light. Furthermore, $I_{45^\circ}$ and $I_{-45^\circ}$ denote the time averaged intensity measured after the light passes a perfect polarizer tilted by $+45^\circ$ and $-45^\circ$, respectively. Similarly, $I_R$ and $I_L$ are the time-averaged intensities of the right-handed and left-handed components of the plane wave. The time-averaged description of the Stokes formalism also allows to take the degree of polarization

$$\begin{align*}
\tan(2\theta) &= \frac{2 \text{Re}(\chi)}{1 - |\chi|^2} \\
\tan(2\psi) &= \frac{2 \text{Im}(\chi)}{1 - |\chi|^2}
\end{align*}$$
\[ \Pi = \sqrt{S_0^2 + S_1^2 + S_2^2} \]  \quad (0 \leq \Pi \leq 1)  \quad (9)

into account. Since every plane wave of the form (2) has a well defined polarization state, the degree of polarization is \( \Pi = 1 \). On the other hand, in the case of unpolarized light, the measurement of the Stokes parameters would lead to \( \Pi = 0 \). This is the typical situation for incoherent light emitted by thermal light sources with rapidly and randomly changing field amplitude vectors. If the degree of polarization is in between the two extremes, the light is partially polarized.

In the case of \( \Pi = 1 \), the Stokes parameters can be related to the quantities in equation (2) in the following way:

\[ S_0 = a^2 + b^2 = \sqrt{S_1^2 + S_2^2 + S_3^2} \]  \quad (10a)

\[ S_1 = a^2 - b^2 = S_0 \cos(2\psi) \cos(2\theta) \]  \quad (10b)

\[ S_2 = 2ab \cos \delta = S_0 \cos(2\psi) \sin(2\theta) \]  \quad (10c)

\[ S_3 = 2ab \sin \delta = S_0 \sin(2\psi) \]  \quad (10d)

Dividing equation (10c) by (10b) and equation (10d) by (10a) yields the angles

\[ \theta = \frac{1}{2} \arctan \left( \frac{S_2}{S_1} \right) \]  \quad (11a)

\[ \psi = \frac{1}{2} \arcsin \left( \frac{S_3}{S_0} \right) . \]  \quad (11b)

In the case of a complex electric field, the Stokes parameters can be derived using

\[ S_0 = E_x E_y^* + E_y E_x^* \]  \quad (12a)

\[ S_1 = E_x E_y^* - E_y E_x^* \]  \quad (12b)

\[ S_2 = E_x E_y^* + E_y E_x^* \]  \quad (12c)

\[ S_3 = i(E_x E_y^* - E_y E_x^*). \]  \quad (12d)

### 2.2. Light propagation in magneto-optic materials

The characteristic property of magneto-optic (MO) materials is that their associated permittivity tensor possesses magnetic field induced off-diagonal elements which are antisymmetric. In this section, it will be elaborated on how this characteristic structure of the permittivity tensor influences the propagation of light inside magneto-optic materials. The most important outcome of this section is that the Faraday effect is directly proportional to the off-diagonal elements in the permittivity tensor.

A key element of the following discussions is the Helmholtz wave equation, which describes the evolution of an electromagnetic wave inside a medium with a given permittivity tensor. At first, this relation is derived from the macroscopic Maxwell equations in matter and then applied to both anisotropic and isotropic MO materials. The derivations in this section are based on the works by Jackson [46], Yariv [47], as well as by Zvezdin and Kotov [44].

#### 2.2.1. Wave equation

Electromagnetic fields in a medium can be described by the macroscopic Maxwell equations

\[ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \] \quad (13a)

\[ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \] \quad (13b)

\[ \nabla \cdot \mathbf{B} = 0 \] \quad (13c)

\[ \nabla \cdot \mathbf{D} = \rho \] \quad (13d)

together with the constitutive equations

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \] \quad (14a)

\[ \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} . \] \quad (14b)

In the following, it is assumed that no sources are present, i.e. \( \mathbf{J} = 0 \) and \( \rho = 0 \). Furthermore, the time-harmonic ansatz

\[ \Psi(\mathbf{r}, t) = \Psi(\mathbf{r}) e^{-i\omega t} \] \quad (15)

turns the equations (13) and (14) into

\[ \nabla \times \mathbf{E}(r) - i\omega \mathbf{B}(r) = 0 \] \quad (16a)

\[ \nabla \times \mathbf{H}(r) + i\omega \mathbf{D}(r) = 0 \] \quad (16b)

\[ \nabla \cdot \mathbf{B}(r) = 0 \] \quad (16c)

\[ \nabla \cdot \mathbf{D}(r) = 0 \] \quad (16d)

and

\[ \mathbf{D}(r) = \varepsilon_0 \varepsilon(\omega) \mathbf{E}(r) \] \quad (17a)

\[ \mathbf{B}(r) = \mu_0 \mu(\omega) \mathbf{H}(r) . \] \quad (17b)

In general, both the relative permittivity \( \varepsilon(\omega) \) and the relative permeability \( \mu(\omega) \) are tensorial quantities and frequency dependent. However, at optical frequencies we can assume that \( \mu = 1 \) [1]. By applying the operator \( \nabla \times \) to equation (16a) and by using the relations \( \nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \Delta \mathbf{E} \) and (16d), the Helmholtz wave equation

\[ \Delta \mathbf{E}(r) + \frac{\omega^2}{\varepsilon_0^2} \varepsilon(\omega) \mathbf{E}(r) = 0 \] \quad (18)

is obtained. This differential equation describes the evolution of an electromagnetic wave inside a medium with a given dielectric tensor \( \varepsilon(\omega) \).

#### 2.2.2. Propagation in anisotropic media

The effect of a static magnetic field on a MO material can be expressed by means of the dielectric tensor of the material. In the following we consider an anisotropic MO material with an applied static magnetic field \( \mathbf{B} \) pointing in \( z \) direction. Furthermore,
we assume that the principal dielectric axes of the material are \( \mathbf{e}_x, \mathbf{e}_y, \) and \( \mathbf{e}_z. \) In this case, the dielectric tensor takes the form
\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & +ig(B) & 0 \\
-ig(B) & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}.
\] (19)

The magnetic-field induced off-diagonal elements of \( \varepsilon \) are the characterizing property of MO materials and the origin for all MO effects, such as the Faraday effect or the MO Kerr effect. The quantity \( g \) is often referred to as gyration coefficient. For small magnetic fields the gyration coefficient is usually proportional to \( B, \) whereas for larger magnetic fields saturation sets in. This is also the case for the materials EuSe and EuS, which are utilized in the MO nanostructures presented in later sections. It should be noted that in the absence of absorption the permittivity tensor must be hermitian, i.e. \( \varepsilon_{ij} = \varepsilon_{ji}^\ast, \) which implies that \( g \) must be real in this case [1].

To solve the wave equation (18) we make the ansatz for a plane wave propagating in \( z \)-direction:
\[
\mathbf{E}(r) = \tilde{\mathbf{E}}e^{ikz}.
\] (20)

Since we can assume that the amplitude vector \( \tilde{\mathbf{E}} \) lies within the \( xy \) plane, we omit its \( z \) component in the following derivations. Hence, we can reduce the permittivity tensor to
\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & +ig & 0 \\
-ig & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}.
\] (21)

Inserting (20) and (21) into the wave equation (18) leads to the eigenvalue problem
\[
\varepsilon \tilde{\mathbf{E}} = \lambda \tilde{\mathbf{E}}
\] (22)

with
\[
\lambda = k^2 \frac{\varepsilon_0}{\omega^2}.
\] (23)

Solving the eigenvalue problem yields the eigenvalues
\[
\lambda_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left( \frac{\varepsilon_x - \varepsilon_y}{2} \right)^2 + \frac{g^2}{4}}
\] (24)

and the eigenvectors
\[
\tilde{\mathbf{E}}^{(1,2)} = \begin{pmatrix}
\frac{\varepsilon_x - \varepsilon_y}{2} \pm \sqrt{\left( \frac{\varepsilon_x - \varepsilon_y}{2} \right)^2 + \frac{g^2}{4}} & 1
\end{pmatrix}.
\] (25)

Consequently, the general solution of (18) is then given by the superposition
\[
\mathbf{E}(r) = A \tilde{\mathbf{E}}^{(1)} e^{ikz} + B \tilde{\mathbf{E}}^{(2)} e^{ikz}
\] (26)

with
\[
k_{1,2} = \frac{\omega}{c_0} \sqrt{\lambda_{1,2}}.
\] (27)

The coefficients \( A \) and \( B \) are determined by the initial condition. For example, the case of \( x \)-polarized incident light implies that \( E_y(z = 0) = 0 \) and \( B = -A, \) which leads to
\[
\mathbf{E}(r) = A \left[ \tilde{\mathbf{E}}^{(1)} e^{ikz} - \tilde{\mathbf{E}}^{(2)} e^{ikz} \right].
\] (28)

Similarly, \( y \)-polarized incident light is described by \( E_z(z = 0) = 0 \) and implies \( B = -A \tilde{E}_{x}^{(1)}/\tilde{E}_{z}^{(2)} \). This leads to a wave propagation of the form
\[
\mathbf{E}(r) = A \left[ \tilde{E}_{x}^{(1)} e^{ikx} - \tilde{E}_{x}^{(2)} e^{ikx} \right].
\] (29)

In both cases, the coefficient \( A \) is an arbitrary normalization parameter depending on the incident intensity. The equations (28) and (29) are utilized in section 5.1 where the magneto-optical response of hybrid plasmonic nanostructures is described by means of a birefringent effective medium.

2.2.3. Faraday rotation. Many bulk magneto-optic materials, such as EuSe and EuS are not birefringent, i.e. the diagonal elements of the dielectric function are equal. In this case, the relative permittivity tensor (21) simplifies to
\[
\varepsilon(\omega) = \begin{pmatrix}
\varepsilon_x & +ig \\
-ig & \varepsilon_x
\end{pmatrix}
\] (30)

and the solution of the eigenvalue problem (22) turns into
\[
\lambda_{1,2} = \varepsilon_x \pm g
\] (31)

and
\[
\tilde{\mathbf{E}}^{(2)} = \begin{pmatrix}
\pm i \\
1
\end{pmatrix}.
\] (32)

Furthermore, the absolute value of the gyration is typically significantly smaller than the diagonal permittivity, i.e. \( |g|/|\varepsilon_x| \ll 1. \) In this approximation, we can write the two propagation constants \( k_1 \) and \( k_2 \) as
\[
k_{1,2} = \frac{\omega}{c_0} \sqrt{\varepsilon_x \pm g} \approx \kappa \mp \gamma,
\] (33)

with
\[
\kappa = \frac{\omega}{c_0} \sqrt{\varepsilon_x}
\] (34)

and
\[
\gamma = -\frac{1}{2} \frac{\omega \varepsilon_x}{c_0}.
\] (35)

From this, through equation (28) for \( x \)-polarized incident light, we obtain
\[
\mathbf{E}(\mathbf{r}, t) = A e^{i(\kappa z - \omega t)} \left[ \begin{pmatrix}
+i \\
1
\end{pmatrix} e^{-i\gamma z} - \begin{pmatrix}
-i \\
1
\end{pmatrix} e^{i\gamma z} \right].
\] (36)

To evaluate how the polarization state of the wave (36) evolves, we can apply the relations (12) to derive the corresponding Stokes parameters
\[
S_0 = +4|A|^2 \cosh (2 \text{ Im } \{\gamma\} z)
\] (37a)

\[
S_1 = +4|A|^2 \cos (2 \text{ Re } \{\gamma\} z)
\] (37b)

\[
S_2 = -4|A|^2 \sin (2 \text{ Re } \{\gamma\} z)
\] (37c)

\[
S_3 = -4|A|^2 \sinh (2 \text{ Im } \{\gamma\} z).
\] (37d)
The Faraday rotation is determined by the direction of the magnetic field.

Furthermore, in the limit of a relatively weak gyration, i.e. for $|\gamma| z \ll 1$, the relations (11) and (35) yield

$$\theta = \frac{1}{2} \frac{\omega}{c_0} \text{Re} \left\{ \frac{g}{\sqrt{\varepsilon}} \right\} z$$

$$\psi = \frac{1}{2} \frac{\omega}{c_0} \text{Im} \left\{ \frac{g}{\sqrt{\varepsilon}} \right\} z. \quad (38a)$$

From this we see that in the case of low material losses and a small imaginary part of $g$, the ellipticity $\psi$ stays small, while the tilting angle $\theta$ increases linearly with the propagation distance $z$. Since the gyration $g$ is proportional to the applied magnetic field $B$, it is very common to write equation (38a) in the alternative form

$$\theta = VBz, \quad (39)$$

where the proportionality factor $V$ is commonly referred to as Verdet constant. However, the term constant is a bit misleading, since $V$ is actually frequency dependent. This is immediately clear by considering that equation (38a) contains both the factor $\omega$ and the gyration $g$, which is also frequency dependent.

The magnetic field induced polarization rotation described by the expression (39) is called Faraday effect and it is illustrated by the blue wave in figure 2. It is important to realize that the magnetic field induced polarization rotation breaks the time-reversal symmetry. The result of this symmetry breaking becomes clear when a mirror is added behind the MO material as indicated in figure 2. In relation to the magnetic field vector, the backward propagating orange light wave is rotated in the same direction and by the same angle as the forward propagating blue wave. Hence, the polarization states of the blue and orange waves on the left-hand side are different. Such a nonreciprocal behavior is not found in other linear and static systems [48]. For example, although an optically active medium [6] can also induce a polarization rotation of transmitted light, any back-reflected wave would rotate back to the polarization state of the incoming wave. In other words, in the case of an optically active medium the direction of polarization rotation is determined by the direction of the wave vector, whereas the direction of Faraday rotation is determined by the direction of the magnetic field.

**2.2.4. Magneto-optic effects in reflection.** The Faraday effect is only one of several phenomena arising from the magnetically induced off-diagonal elements in $\epsilon(\omega)$. For example, also the reflection behavior of a MO material can change depending on the applied magnetic field. This phenomenon is referred to as magneto-optic Kerr effect (MOKE). Since the MOKE does rely on reflection rather than propagation it cannot only be observed for transparent materials but also for opaque materials such as ferromagnetic metals.

Although the work presented in this review concentrates on the Faraday effect, in the following a brief overview of the different types of the MOKE is given. As illustrated in figure 3, the different types are classified depending on the direction of the magnetization in relation to the plane of incidence and the material surface [44]. The black arrows indicate the wavevectors of the incident and reflected light. If the magnetization is perpendicular to the material surface, there occurs a magnetically induced polarization rotation as well as a change in ellipticity of the reflected light. This effect is referred to as the polar MOKE. Similarly, if the magnetization is parallel to the plane of incidence, the intensity of the reflected light changes with the magnitude of magnetization. This phenomenon is called transverse MOKE.

Since the polarization rotation of the MOKE only scales with the off-diagonal elements of the permittivity tensor, the MOKE is generally much weaker than the Faraday effect, which also scales with the optical path length through the material. For example, although Fe, Co and Ni possess an
Lorentz force is given by $\mathbf{F}_L = q B (\dot{y}, -\dot{x}, 0)^T$ and the resulting equations of motion are

$$
\begin{align*}
\dot{m}x &= -d_x x - 2\gamma_x \dot{x} + q B \dot{y} + q E_x \exp(-i\omega t) \tag{41a} \\
\dot{m}y &= -d_y y - 2\gamma_y \dot{y} - q B \dot{x} + q E_y \exp(-i\omega t) \tag{41b} \\
\dot{m}z &= -d_z z - 2\gamma_z \dot{z} + q E_z \exp(-i\omega t). \tag{41c}
\end{align*}
$$

The quantities $\{d_x, d_y, d_z\}$ and $\{\gamma_x, \gamma_y, \gamma_z\}$ denote the stiffness and damping coefficients of the springs. To solve the equations, we start by making a time harmonic ansatz for the oscillator displacement, i.e.

$$
\mathbf{r}(t) = \mathbf{r}_0 \exp(-i\omega t), \tag{42}
$$

which turns the equations (41) into

$$
\begin{align*}
(-\omega^2 + \Omega_x^2 - 2i\Gamma_x\omega)x_0 + i\omega\omega_c y_0 &= \frac{q}{m} E_x \tag{43a} \\
(-\omega^2 + \Omega_y^2 - 2i\Gamma_y\omega)y_0 - i\omega\omega_c x_0 &= \frac{q}{m} E_y \tag{43b} \\
(-\omega^2 + \Omega_z^2 - 2i\Gamma_z\omega)z_0 &= \frac{q}{m} E_z. \tag{43c}
\end{align*}
$$

with $\Omega_i^2 = d_i/m$ and $\Gamma_i = \gamma_i/m$ ($i = x, y, z$) and $\omega_c = qB/m$. The equations (43) can be reformulated in the matrix form, which yields

$$
M(\omega) \mathbf{r}_0 = \frac{q}{m} \mathbf{E}, \tag{44}
$$

with

$$
M(\omega) = \begin{pmatrix} M_x & +i\omega\omega_c & 0 \\ -i\omega\omega_c & M_y & 0 \\ 0 & 0 & M_z \end{pmatrix}, \tag{45}
$$

and $M_{ix}(\omega) = -\omega^2 + \Omega_i^2 - 2i\Gamma_i\omega$ ($i = x, y, z$). Assuming that $n$ is the oscillator density in the medium, we can express the macroscopic electronic polarization as

$$
\mathbf{P} = n q \mathbf{r}_0 = \frac{n q^2}{m} M^{-1} \mathbf{E}. \tag{46}
$$

Furthermore, the comparison with

$$
\mathbf{P} = \varepsilon_0 \chi \mathbf{E}, \tag{47}
$$

allows to identify the electronic susceptibility

$$
\chi(\omega) = \omega_q^2 M(\omega)^{-1}, \tag{48}
$$

with

$$
\omega_q = \sqrt{\frac{n q^2}{\varepsilon_0 m}}. \tag{49}
$$

The optical response of a material is usually expressed by the electric permittivity tensor $\varepsilon = 1 + \chi$, where $\chi$ denotes the identity matrix $\text{diag}(1, 1, 1)$. To also account for a (scalar) background susceptibility due to other off-resonant electronic polarizations [52], we write the permittivity in the more general form $\varepsilon = \varepsilon_\infty 1 + \chi$, where $\varepsilon_\infty$ corresponds to the value of the diagonal permittivity elements for infinite frequency. With that, we arrive at
\[ \varepsilon(\omega) = \varepsilon_\infty + \omega^2 M(\omega)^{-1}. \]  

(50)

In the anisotropic case, i.e. when the stiffness and damping coefficients of all oscillators are different, the inverse of \( M(\omega) \) and thus also the permittivity tensor \( \varepsilon(\omega) \) exhibit a very complicated \( \omega \) dependence. However, many magneto-optical materials are isotropic, that is, \( \Omega_x = \Omega_y = \Omega_z = \Omega \) and \( \Gamma_x = \Gamma_y = \Gamma_z = \Gamma \). In this case we obtain

\[
\varepsilon = \begin{pmatrix}
\varepsilon_{11} & \varepsilon_{12} & 0 \\
\varepsilon_{21} & \varepsilon_{22} & 0 \\
0 & 0 & \varepsilon_{33}
\end{pmatrix}
\]  

(51)

with

\[
\varepsilon_{11} = \varepsilon_{22} = \frac{\omega^2 (-\omega^2 + \Omega^2 - 2i\Gamma_\omega)}{(-\omega^2 + \Omega^2 - 2i\Gamma_\omega)^2 - \omega^2 \omega_i^2} + \varepsilon_\infty
\]  

(52a)

\[
\varepsilon_{12} = -\varepsilon_{21} = \frac{-i\omega \omega_i \omega_i}{(-\omega^2 + \Omega^2 - 2i\Gamma_\omega)^2 - \omega^2 \omega_i^2}
\]  

(52b)

\[
\varepsilon_{33} = \frac{\omega^2}{-\omega^2 + \Omega^2 - 2i\Gamma_\omega} + \varepsilon_\infty.
\]  

(52c)

From the equations (52) we can see that the magnetic field not only influences the off-diagonal elements of the dielectric function but also the diagonal elements, which can be regarded as the classical manifestation of the Zeeman effect [51]: In the case of low damping (\( \Gamma \) is small), the resonance condition for \( \varepsilon_{11} \) is given by

\[
\omega \approx \Omega \pm \frac{\omega_i}{2},
\]  

(53)

which means that the material resonance is split by the magnetic field.

However, in most magneto-optical applications, the influence of the magnetic field on the diagonal components can be neglected, since usually \( |\omega_i| \ll \omega, \Omega, \Gamma \). In this approximation the components of \( \varepsilon \) reduce to

\[
\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = \frac{\omega^2}{-\omega^2 + \Omega^2 - 2i\Gamma_\omega} + \varepsilon_\infty
\]  

(54a)

\[
\varepsilon_{12} = -\varepsilon_{21} = \frac{-i\omega \omega_i \omega_i}{(-\omega^2 + \Omega^2 - 2i\Gamma_\omega)^2}.
\]  

(54b)

The model function (54) provides a very good qualitative description of the magneto-optic response of many materials. There are several important properties of the model: firstly, since \( \omega_i \) is proportional to the magnetic field, the model correctly predicts a linear relation between the off-diagonal elements of the permittivity and the magnetic field. In the regime of weak magnetic fields, this is the case for most magneto-optic materials. Furthermore, the model provides a Kramers–Kronig-consistent relation between the real and imaginary part of \( \varepsilon_{12} \) [44].

The permittivity function can be easily implemented in many numerical simulation tools. However, to obtain quantitatively correct simulation results, the free model parameters in the relations (54) have to be fitted to measurement data. For some MO materials it can be necessary to add further oscillators to the polarization (46) with different individual parameter sets, in order to account for a more complex MO behavior.

### 2.3.2. Quantum mechanical description

Compared to the simple mechanical model provided in section 2.3.1 the advantage of the following quantum mechanical description is that it can explain different dispersion behaviors of the magneto-optic spectra (i.e. diamagnetic and paramagnetic lineshapes) as well as the influence of temperature on the magnitude of the Faraday rotation. The following discussion is derived from [44] and [53].

To understand the origin of Faraday rotation in atomic systems, we will make use of the fact that Faraday rotation can be regarded as a manifestation of magnetic circular dichroism (MCD), i.e. different absorption of right-handed circularly polarized (RCP) and left-handed circularly polarized (LCP) light [44]. The reason is that it is much simpler to relate atomic properties to MCD than to the Faraday effect directly. We note in passing, that, as in the case of optical activity [6, 54], polarization rotation and circular dichroism are connected via Kramers–Kronig relations [44].

In magneto-optics it is common to distinguish two contributions to MCD: firstly, due to the Zeeman effect, atomic levels are split into several levels that can be excited only with light of the correct handedness. This results in absorption maxima of LCP and RCP light which are slightly split in energy. As a consequence, the absorption of LCP and RCP is different for most wavelengths. This mechanism is referred to as diamagnetic Faraday rotation, as it often occurs in diamagnetic materials. The second mechanism contributing to MCD (if present) usually exceeds the diamagnetic contribution: due to temperature or other influences, the effective oscillator strength of either the LCP or RCP transition can be suppressed. This can dramatically increase the absorption difference of LCP and RCP and thus the Faraday rotation. This mechanism is called paramagnetic Faraday rotation as paramagnetic materials often exhibit such a behavior. At this point it should be emphasized that the terms diamagnetic and paramagnetic are not related to the magnetic susceptibility but are part of an established naming scheme for classifying the dispersion in magneto-optical spectra [44].

A more detailed comparison between the diamagnetic and paramagnetic mechanisms is illustrated in the figures 5(a) and (b) respectively. In the diamagnetic case the ellipticity spectrum is S-shaped (orange line) whereas the Faraday rotation shows a peak surrounded by two smaller negative peaks (blue line). This behavior is very similar to chiral media [54]. The lower part of figure 5(a) shows an exemplary and idealized energy level scheme which would result in such a magneto-optic dispersion. In this example we consider a transition from a \(^1S_0\) state to a \(^1P_1\) state. Due to the magnetic field the \(^1P_1\) state is split into three states with quantum numbers \( M_J = 0, +1, -1 \). An incoming LCP or RCP photon induces a transition with \( \Delta M = +1 \) or \( \Delta M = -1 \), respectively. Due to the magnetic splitting, the absorption lines of the LCP and RCP transitions (denoted by \( \sigma_+ \) and \( \sigma_- \)) are shifted in energy. Thus, the absorption for LCP and RCP light is different for
most wavelengths and results in the depicted Faraday rotation dispersion. We note that this is in fact the dispersion behavior as predicted by the mechanical oscillator description in section 2.3.1.

A different behavior can be observed in the paramagnetic case, which is schematically depicted in figure 5(b). This situation arises when the oscillator strengths of the LCP and RCP transitions is significantly different. To illustrate this, in the lower part of figure 5(b) an idealized energy level diagram is depicted. The transition energies and the magnetic field induced splitting are assumed to be the same as in the diamagnetic example. Hence, there should also be a diamagnetic contribution to the Faraday rotation spectrum with the same magnitude. However, here the usually much stronger paramagnetic mechanism dominates the magneto-optical spectra: For $T \to 0$ K the population probability of the ($M_J = -1$) level is much higher than for the ($M_J = +1$) level, resulting in a magnetization. Therefore, the $\sigma_+$ transition dominates. This difference in oscillator strength is responsible for the paramagnetic Faraday rotation dispersion.

The absolute magnitude of the magneto-optical response depends on the specific atomic structure of the atoms in the medium. In the case of solids, the resulting band structure is critical. There are many factors that contribute to a strong magneto-optic response and also to a dispersion behavior deviating from the two idealized cases discussed above. This includes spin-orbit interactions, exchange interactions, and further effects. A more detailed compilation of possible influences is left to specialized literature, such as [44, 53, 55, 56]. However, in the following it is motivated why spin-orbit interaction is one of the most important ingredients for a large Faraday rotation response (as is the case for Eu compounds).

To understand the influence of spin-orbit coupling, we consider the exemplary dipole transitions depicted in figure 6. The diagram shows transitions from a $^2S_{1/2}$ state to $^2P$ states. Due to the spin-orbit interaction the $^2P$ state is split into the two levels $^2P_{1/2}$ ($J = 1/2$) and $^2P_{3/2}$ ($J = 3/2$). The presence of a magnetic field lifts the degeneracy of levels with the same total angular momentum. The arrows indicate the dipole allowed transitions which fulfill the selection rule $\Delta M_J = \pm 1$. It can be shown [44] that the non-zero transition dipole moments $d_{ab}^{\pm}$ fulfill the relations

$$|d_{14}^+|^2 = |d_{33}^+|^2 = (2/9)d^2$$  
$$|d_{14}^-|^2 = |d_{28}^+|^2 = (1/3)d^2$$  
$$|d_{17}^+|^2 = |d_{28}^-|^2 = (1/9)d^2$$

where $d$ is constant. For $T \to 0$ only level 1 is populated and only the transitions to the levels 4, 5 and 7 are possible. Neglecting the difference in transition frequency, their contribution to MCD (and thus Faraday rotation) would cancel out, since from equation (55) follows $|d_{14}^+|^2 - |d_{15}^-|^2 + |d_{17}^+|^2 = 0$. This means that the Faraday rotation occurs only due to the different transition frequencies, as it was the case in the diamagnetic mechanism discussed above. It can be seen directly from figure 6 that, with increasing spin-orbit splitting, the difference in the transition frequencies become larger and thus the overall Faraday rotation becomes larger. When the temperature increases, the population of level 2 starts to grow and also contributes to the Faraday rotation. In the high temperature limit, the populations of level 1 and 2 are the same and it can be shown that the overall contribution to the Faraday rotation by the transitions from level 2 is of the same magnitude as the contribution by the transitions from level 1, yet with opposite sign [44]. More precisely, the Faraday rotation is proportional to the population difference between level 1 and 2, i.e. to the average magnetic moment of the atom or ion (as in the discussion of the paramagnetic mechanism). In summary, we have seen that Faraday rotation tends to decrease with higher temperatures (and lower average magnetic moment) and strong spin-orbit interaction can be beneficial for large rotation angles.

2.4. Comparison of typical magneto-optic materials

Materials with a particularly strong magneto-optic response are usually referred to as magneto-optic (MO) materials. The general properties of these materials differ strongly, not only with respect to the magnitude of the MO response, but also in...
many other aspects. For example, there are metallic as well as non-metallic MO materials, MO materials with different magnetic susceptibility, as well as transparent and opaque materials (depending on wavelength). Furthermore, the MO properties usually strongly depend on temperature. In this section, some of the most relevant MO materials are compared. However, it should be mentioned that the collection of materials provided here is not complete, especially because there are many doped variants of the listed materials as well as materials with slightly modified chemical composition. An extended overview can be found in more specialized literature [44, 53, 56–60].

Figure 7 gives a rough overview of the range of available MO materials and the magnitude of the achievable MO response in terms of Faraday rotation per unit length. The data was extracted from [55]. The graph shows that the materials with the largest specific Faraday rotation response are ferromagnetic metals such as Fe, Co, and Ni. Also quite strong Faraday rotation can be achieved by using Europium compounds at low temperatures such as EuSe or by CrI₃, CrBr₃ and MnBi. EuS, EuTe and EuO are not shown in the diagram but exhibit optical and MO properties similar to EuSe [53, 58]. Relatively weak Faraday rotation is obtained by yttrium–iron–garnet (YIG) as well as by CrCl₃ at shorter wavelengths. We should note at this point that although the spectrum of YIG is plotted for low temperatures [55] the material also exhibits Faraday rotation of comparable magnitude at room temperature [61].

In many cases the specific Faraday rotation is not a suitable figure of merit for the performance of a MO material. Especially, when a MO material is to be used in transmission geometry, the absorption of the material is relevant. Thus, a more useful way to characterize MO materials can be the specific Faraday rotation normalized to attenuation (in dB cm⁻¹) shown in figure 8 (data from [55]). This graph reveals that ferromagnetic metals, which have by far the largest MO response, also exhibit the largest optical losses. Although their room temperature compatibility is convenient, their opacity makes them only useful for MO effects in reflection geometry (e.g. for the MO Kerr effect). In contrast, EuSe exhibits both high MO response and high transparency. This is also the case for other Eu compounds such as EuS, EuTe, and EuO, which are not plotted here [53, 58]. Although the ratio of Faraday rotation and attenuation can be a meaningful quantity, it should not be regarded as an universal figure of merit for MO applications. For example, in the near IR YIG exhibits a very high ratio of Faraday rotation and attenuation. However, the absolute Faraday rotation in this frequency region is extremely weak (see figure 7). This means that a polarization rotation device made of YIG would require a relatively large amount of MO material in order to achieve a sizable rotation angle. Hence, for realizing a small scale Faraday rotator, an Eu compound would be more suitable, yet at lower temperatures.

In state of the art nanoscale MO systems, there are three predominant groups of utilized MO materials: metals [21, 30–33], iron garnets [34, 36, 62, 63] and Eu compounds (sections 4 and 6). Due to the high optical losses of metals, the first group is mainly used for systems that operate in reflection geometry (i.e. for utilizing the MO Kerr effect). In the following, we focus on the comparison between iron garnets and Eu compounds, which are suitable for the utilization in Faraday geometry.

Since garnets possess a relatively low Verdet constant, there have been efforts to modify their chemical structure in
order to enhance their MO response. For example, in the case of YIG, substituting yttrium with bismuth leads to a greatly increased Faraday effect [60]. However, the relatively large Faraday rotation of bismuth–iron–garnet (BIG) comes with significant fabrication difficulties: BIG only forms in sophisticated non-equilibrium processes, which involve pulsed laser deposition as well as a subsequent high temperature annealing. This makes the fabrication of hybrid systems incorporating plasmonic nanostructures very challenging and restricts the number of possible structure geometries considerably. For example, any gold nanostructures that are incorporated inside a BIG film would get damaged due to the high temperatures. Moreover, BIG only grows on garnet substrates or special buffer layers. However, solutions to overcome these problems such as coating with a few nm of Al₂O₃ or SiO₂ by evaporation or atomic layer deposition might be possible [64, 65].

Eu compounds such as EuSe and EuS provide very simple fabrication by physical vapor deposition and are also compatible with standard electron beam lithography processes. As such, these materials can be used to realize sophisticated layer based nanostructure geometries consisting of both magneto-optic an plasmonic materials. EuSe and EuS possess very strong MO response below and around their Curie temperatures at 7 K and 16.6 K, respectively.

For the sake of completeness, it should be mentioned that outside of the visible and near infrared regime there is a higher availability of materials with both high Verdet constant and reasonable transparency. For example, in 2011 Shuvaev et al showed that in the THz regime HgTe can provide 14° Faraday rotation already for a 70 nm thick film at 1 T [66]. Furthermore, Crassee et al demonstrated in 2010 that at low temperatures a single layer of graphene can produce Faraday rotation on the order of 5° in the THz regime for a magnetic field of 7 T [67].

2.5. Plasmonics

In this section the basic optical properties of metal interfaces and nanostructures are summarized. This allows us to understand the emergence of localized surface plasmon resonances in gold nanostructures as used in the following sections. It will turn that plasmonic resonances are of fundamental importance for tailoring the dispersion properties of hybrid magnetoplasmonic systems.

This section begins with a discussion of the general optical properties of metals based on the plasma model (also known as Drude model). This is the foundation for the subsequent analysis of the two most prominent plasmonic phenomena: surface plasmon polaritons and localized surface plasmon resonances. As this section cannot provide full coverage of all aspects of plasmonics and related electrodynamic effects, the interested reader can find very detailed further discussions in the works [46, 68], which this section is based on.

2.5.1. The dielectric function of metals. The characterizing optical properties of metals can be derived from the plasma model, also known as the Drude model. For many metals the validity of this simple model extends over a surprisingly wide wavelength range. The basic assumption of the model is that the unbound electrons with number density \( n \) are moving freely against the fixed and positively charged ions. This means that the electrons are not subjected to a restoring force as in the Lorentz oscillator model discussed in section 2.3.1. However, as in the Lorentz model, the electrons are assumed to be driven by the time harmonic electric field \( \mathbf{E}(t) = E_0 e^{-i\omega t} \) of the light wave. The resulting equation of motion for the electron displacement \( \mathbf{x}(t) \) is given by

\[
m \ddot{x}(t) + m \gamma \dot{x}(t) = -e_0 E_0 e^{-i\omega t},
\]

where \( m \) and \(-e_0\) are the effective mass and charge of an individual electron. Furthermore, the damping constant \( \gamma = 1/\tau \) corresponds to the characteristic collision frequency of a plasma electron. The differential equation (58) can be solved by the time harmonic ansatz

\[
x(t) = x_0 e^{-i\omega t},
\]

which leads to the solution

\[
x(t) = \frac{e_0}{m(\omega^2 + i\gamma \omega)} E(t).
\]

Furthermore, by means of equation (60), the macroscopic polarization of the plasma \( \mathbf{P} = -ne_0 \mathbf{x} \) can be written as

\[
\mathbf{P} = \frac{-ne_0^2}{m(\omega^2 + i\gamma \omega)} \mathbf{E}.
\]

With equation (61) we have now established the relation between the polarization of the plasma and the applied electric field. Hence, via the relation \( \mathbf{P} = e_0 \chi(\omega) \mathbf{E} \), the electric susceptibility \( \chi(\omega) \) can be identified as

\[
\chi(\omega) = \frac{-ne_0^2}{me_0(\omega^2 + i\gamma \omega)}.
\]

Finally, by introducing the plasma frequency \( \omega_p \)

\[
\omega_p = \frac{ne_0^2}{me_0}
\]

and by using \( \varepsilon(\omega) = 1 - \chi(\omega) \), the complex dielectric function of the free electron gas can be written as

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma \omega}.
\]

The real and imaginary parts of \( \varepsilon(\omega) \) are given by

\[
\text{Re}[\varepsilon(\omega)] = 1 - \frac{\omega_p^2 \tau^2}{1 + \omega^2 \tau^2}
\]

\[
\text{Im}[\varepsilon(\omega)] = \frac{\omega_p^2 \tau}{\omega + \omega^2 \tau^2}.
\]

In order to increase the agreement between the modeled permittivity (64) and the one of real metals, it can be useful to introduce a slight modification. By adding a constant parameter to the expression (64), the contribution of the positive metal ions to the overall polarization can be taken into account [68]. With this modification, the dielectric function becomes
\[ \sigma = -n e_0 d \]

where \( \sigma \) is the electrical conductivity.

\[ \varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma \omega}, \quad (67) \]

Figure 9. Illustration of a volume plasmon. The electrons in the metal are collectively driven at the plasma frequency \( \omega_p \). They are displaced by the distance \( d \) relative to positively charged ions. This results in a surface charge density \( \sigma = -n e_0 d \).

\[ \varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma \omega}, \quad (68) \]

For the propagating electromagnetic wave we make the ansatz

\[ \mathbf{E}(z,t) = E_0 e^{i(kz - \omega t)}. \quad (69) \]

By inserting the equations (68) and (69) into the Helmholtz wave equation (18) the dispersion relation

\[ k^2 = \frac{\omega^2}{c^2} \varepsilon(\omega) = \frac{\omega^2 - \omega_p^2}{c^2}, \quad (70) \]

is obtained. It is evident that there are two distinct frequency regimes: for \( \omega < \omega_p \), the propagation constant \( k \) becomes imaginary, i.e. there is no light propagation possible. On the other hand, for \( \omega > \omega_p \), transverse electromagnetic waves can propagate through the plasma, as \( k \) is real.

A physically interesting situation occurs if \( \omega = \omega_p \). In this case, the propagation constant \( k \) vanishes, i.e. the electron excitation becomes a collective movement. Furthermore, the dielectric function \( \varepsilon(\omega_p) \) becomes zero. Together with \( \mathbf{D} = \varepsilon_0 \varepsilon(\omega_p) \mathbf{E} = 0 \) the equation (14) yields

\[ \mathbf{E} = -\mathbf{P}/\varepsilon_0, \quad (71) \]

which means that the electric field is purely originating from the polarization of the medium. Figure 9 illustrates how this situation can be interpreted in the case of a flat piece of metal. The electrons are collectively displaced by a distance \( d \) leading to the surface charges \( \sigma = -n e_0 d \) and \( \sigma = +n e_0 d \) on the two sides of the slab. The attractive force between the two charged surfaces normalized to a single electron can be written as \( F = md = -e_0 |\mathbf{E}| \) with \( |\mathbf{E}| = ne_0d/\varepsilon_0 \). This results in the equation of motion

\[ \ddot{d}(t) + \omega_p^2 d(t) = 0. \quad (72) \]

From this follows that the plasma frequency can be interpreted as the characteristic frequency of a collective electron gas that is oscillating relatively to a fixed positively charged background. The excitation of such an oscillation is referred to as volume plasmon. At this point it should be noted that volume plasmons are a fundamentally different type of excitation compared to surface plasmons and localized surface plasmons, which is discussed in the next sections. Volume plasmons are of longitudinal nature and can thus not couple to transverse light waves. Thus, they do not play a role for the magnetoplasmonic systems discussed in the following sections.

Figure 10 depicts a comparison of the Drude model function (67) (orange curves) fitted to the measured permittivity of gold (blue dots). For low frequencies there is a good agreement between the modeled permittivity and the measurement. However, for higher frequencies interband transitions cause a significant deviation from the model. The measurement data was extracted from [68].
to a restoring force in analogy to the Lorentz oscillators discussed in section 2.3.1.

2.5.2. Surface plasmon polaritons. Surface plasmon polaritons (SPPs) are electromagnetic waves propagating along metal-dielectric interfaces. Although SPPs are not directly utilized in the magnetoplasmonic systems analyzed in later sections, the discussion of their most important properties provides an important context for the analysis of localized surface plasmon resonances presented in the next section.

In the following, the electrodynamics of a SPP are analyzed for the simplest case of a flat metal-dielectric interface at \( z = 0 \), as illustrated in figure 11. For an electromagnetic wave propagating in \( x \) direction we make the ansatz

\[
E(r, t) = E(z) e^{i(\beta x - \omega t)} \quad \text{(73a)}
\]

\[
H(r, t) = H(z) e^{i(\beta x - \omega t)}. \quad \text{(73b)}
\]

Since the geometry is invariant in \( y \) direction, \( H(z) \) and \( E(z) \) can be assumed to be independent of \( y \). By inserting (73) into Maxwell’s equations for time harmonic fields (16) it can be shown that there are two classes of solutions: TE-polarized waves, with only the components \( H_y, H_z \) and \( E_y \) being non-zero and TM-polarized waves, with only the components \( E_x, E_z \) and \( H_y \) being non-zero. However, it should be noted that it can be shown that TE waves do not fulfill the condition that \( E_y \) and \( D_z \) are continuous at the metal-dielectric interface [68]. Hence, SPPs are always TM polarized. In the case of the TM waves, the Maxwell equations (16) yield

\[
\frac{\partial^2}{\partial z^2} H_y(z) + \left[ \frac{\omega^2}{c_0^2} \epsilon(\omega, z) - \beta^2 \right] H_y(z) = 0 \quad \text{(74a)}
\]

\[
E_x(z) = \frac{1}{i \omega \epsilon_0 \epsilon(\omega, z)} \frac{\partial H_y(z)}{\partial z} \quad \text{(74b)}
\]

\[
E_z(z) = -\frac{\beta}{\omega \epsilon_0 \epsilon(\omega, z)} H_y(z). \quad \text{(74c)}
\]

Within the dielectric half-space \( (z > 0) \) the solutions of the equations (74) are given by

\[
H_y(z) = C_d e^{-\kappa_d z} \quad \text{(75a)}
\]

\[
E_z(z) = C_d e^{-\kappa_d z} \quad \text{(75b)}
\]

\[
E_x(z) = C_d e^{-\kappa_d z} \frac{\kappa_d}{\omega \epsilon_0 \epsilon_d} (-1). \quad \text{(75c)}
\]

where the positive decay constant \( \kappa_d \) is to obey

\[
\kappa_d^2 = \beta^2 - \frac{\omega^2}{c_0^2} \epsilon_d. \quad \text{(76)}
\]

Similary, for \( z < 0 \) the solutions are given by

\[
H_y(z) = C_m e^{+\kappa_m z} \quad \text{(77a)}
\]

\[
E_x(z) = C_m e^{+\kappa_m z} \frac{\kappa_m}{\omega \epsilon_0 \epsilon_m} (-1) \quad \text{(77b)}
\]

\[
E_z(z) = C_m e^{+\kappa_m z} \frac{\beta}{\omega \epsilon_0 \epsilon_m} (-1) \quad \text{(77c)}
\]

with the condition

\[
\kappa_m^2 = \beta^2 - \frac{\omega^2}{c_0^2} \epsilon_m. \quad \text{(78)}
\]

for decay constant \( \kappa_m \), which is also positive. At the interface between the two half-spaces the normal component of \( \mathbf{D} = \epsilon_0 \mathbf{E} \) and the tangential component of \( \mathbf{E} \) have to be continuous. From this follows that \( C_m = C_d \) and

\[
\frac{\kappa_m}{\kappa_d} = -\frac{\epsilon_m}{\epsilon_d}. \quad \text{(79)}
\]

Equation (79) is an important intermediate result. We see that modal confinement \( (\kappa_m, \kappa_d > 0) \) requires \( \text{Re}(\epsilon_m) < 0 \), if \( \epsilon_d > 0 \). In other words, SPPs indeed occur only at metal-dielectric interfaces. Furthermore, we can use equation (79) to

![Figure 11](image1.png) Illustration of a surface plasmon polariton (SPP) propagating along a flat metal-dielectric interface at \( z = 0 \).

![Figure 12](image2.png) Dispersion diagram for surface plasmon polaritons at a metal-air interface (blue) and a metal-glass interface (orange). Solid lines correspond to the real part of the wavenumber \( \beta \), whereas the dotted lines denote the imaginary part. The dash-dotted lines indicate the corresponding light cones of the dielectric half-spaces.
eliminate $\kappa_m$ and $\kappa_d$ in the equations (76) and (78) and obtain the dispersion relation of SPPs:

$$\beta(\omega) = \frac{\omega}{c_0} \sqrt{\frac{\varepsilon_m(\omega)\varepsilon_d(\omega)}{\varepsilon_m(\omega) + \varepsilon_d(\omega)}}. \quad (80)$$

For the sake of simplicity, the dispersion of SPPs is discussed for the case of a perfect Drude metal without damping, i.e. it is assumed that the dielectric function of the metal half-space to be (64) with $\gamma = 0$. Figure 12 displays the plots of $\text{Re}(\beta)$ (solid lines) and $\text{Im}(\beta)$ (dotted lines) for a metal-air interface (blue) and a metal-glass interface (orange). The light cones of the corresponding dielectric half-spaces are indicated by dash-dotted lines. The dielectric function of glass is assumed to be $\varepsilon_d = 2.3$. The plots show that the SPP wavenumber is purely imaginary between the plasma frequency and the characteristic surface plasmon frequency $\omega_{sp} = \omega_p / \sqrt{1 + \varepsilon_d}$. Hence, the SPP propagation in this range is prohibited. On the other hand, for $\omega < \omega_{sp}$ the SPP wavenumber is purely real and the propagation of SPP waves is possible. For $\omega > \omega_p$ the metal becomes transparent (see section 2.5.1). Since the dispersion curves of the propagating SPPs lie outside of the light cones of the dielectric materials (dash-dotted lines), these waves cannot be excited by incident plane waves, as the photon energy and momentum cannot be conserved simultaneously. However, there is a variety of techniques that allow to excite SPPs by an incident light beam. Typical examples involve prism couplers, grating couplers and other surface perturbations such as edges or even dust particles. Finally, it should be pointed out that the dielectric function of a real metal does involve significant absorptive contributions, which result in a relatively fast decay of the propagating SPPs. Typical propagation lengths of SPPs are on the order of 10–100 $\mu$m [68].

2.5.3. Localized surface plasmon resonances. In this section the second type of plasmonic excitations will be introduced: localized surface plasmon resonances (LSPRs). This plasmonic phenomenon is of fundamental relevance for the magnetoplasmonic structures presented in later sections. In contrast to the surface plasmon polaritons discussed in the previous section, LSPRs are non-propagating excitations. They occur in sub-wavelength metal nanoparticles surrounded by a dielectric medium. Typical resonance wavelengths of gold and silver particles are in the visible and near infrared. The small particle size also results in a high surface curvature, which enables the direct excitation of LSPRs by plane electromagnetic waves. It should be pointed out that LSPRs not only occur in metal volumes which are confined in all three dimensions, but also in structures which are infinitely extended in one dimension (i.e. in metal wires as utilized in the following sections). In that case, the localized oscillation takes place across the wire, perpendicularly to the extended wire axis.

To understand the fundamental mechanism behind a LSPR it is useful to investigate the simple case of a spherical metal particle with a radius $a$ well below the wavelength of light. This situation is illustrated in figure 13. Due to the small particle size, the electric field of a surrounding electromagnetic wave can be assumed to be constant within a volume of the size of the sphere. Hence, the electric field of the light wave can be treated as a static field $E_0 = E_0 e_z$ and the system is described by the Laplace equation

$$\Delta \Phi(r) = 0,$$  \hspace{1cm} (81)

where the electric field is obtained via $E(r) = -\nabla \Phi(r)$. Any time-harmonic dependence can then be added subsequently to the static solution, i.e. $E(r, t) = E(r) e^{-i\omega t}$. This assumption is known as quasi-static approximation. Since the geometry in figure 13 possesses revolution symmetry around the $z$-axis we can write the general solution of (81) as

$$\Phi(r, \theta) = \sum_{l=0}^{\infty} \left[ A_l r^l + B_l r^{-(l+1)} \right] P_l(\cos \theta),$$  \hspace{1cm} (82)

where $P_l$ are the Legendre polynomials [46]. The coefficients $A_l$ and $B_l$ are determined by the following conditions: firstly, at the surface of the metal sphere, the tangential component of $E(r)$ and the normal component of $D(r) = \varepsilon_0 \varepsilon(\omega) E(r)$ have to be continuous. Additionally, it is required that $\Phi(r, \theta) \to E_0 r \cos \theta$ as $r \to \infty$. Applying these conditions results in

$$\Phi(r, \theta) = -\frac{3\varepsilon_d}{\varepsilon + 2\varepsilon_d} E_0 r \cos(\theta)(r < a)$$  \hspace{1cm} (83a)

$$\Phi(r, \theta) = -E_0 r \cos(\theta) + \frac{\varepsilon - \varepsilon_d a^3}{\varepsilon + 2\varepsilon_d} E_0 \frac{\cos(\theta)}{r^2} (r > a).$$  \hspace{1cm} (83b)

By comparing the second term in equation (83b) to the generic potential of a dipole in $z$ direction [46]

$$\Phi(r, \theta) = -\frac{1}{4\pi \varepsilon_0 \varepsilon_d} \frac{p \cos(\theta)}{r^2}$$  \hspace{1cm} (84)

we can identify

$$p = 4\pi \varepsilon_0 \varepsilon_d \frac{\varepsilon - \varepsilon_d}{\varepsilon + 2\varepsilon_d} a^3 E_0.$$  \hspace{1cm} (85)

Furthermore, by using $E_0 = E_0 e_z$ and $r = re_z$, we can express the equations (83) in the form...

---

**Figure 13.** Illustration of a spherical metal nanoparticle surrounded by a dielectric medium. If the radius $a$ is well below the wavelength of light, the condition for the localized surface plasmon resonance can be obtained by means of the quasi-static approximation.
Hence, by introducing the polarizability \( \alpha(\omega) \) via the relation \( \mathbf{p} = \varepsilon_0 \varepsilon_d \alpha(\omega) \mathbf{E}_0 \), equation (85) yields the important result

\[
\alpha(\omega) = 4\pi a^3 \frac{\varepsilon(\omega) - \varepsilon_d}{\varepsilon(\omega) + 2\varepsilon_d}.
\]

We see immediately that when \( \varepsilon(\omega) \) approaches \( -2\varepsilon_d \) the polarizability exhibits a resonant behavior. In the case of an approximately constant \( \text{Im}[\varepsilon(\omega)] \) around that resonance, the resonance condition becomes

\[
\text{Re} \left[ \varepsilon(\omega) \right] = -2\varepsilon_d,
\]

which is known as the Fröhlich condition. For a nanoparticle consisting of an ideal Drude metal without damping (with a dielectric function (68)) the Fröhlich condition condition is met at the resonance frequency

\[
\omega_{\text{SPR}} = \frac{\omega_p}{\sqrt{1 + 2\varepsilon_d}}.
\]

From this we see that an increase of the refractive index around the metal particle causes a red-shift of the resonance. This behavior is utilized in refractive index sensing [69]. With the resonantly increased polarizability (and thus light absorption) also comes a field enhancement inside and in the vicinity of the nanoparticle. This can be immediately recognized by deriving the electric fields \( \mathbf{E} = -\nabla \Phi \) associated with the potentials (86):

\[
\Phi(\mathbf{r}) = -\frac{3\varepsilon_d}{\varepsilon + 2\varepsilon_d} \mathbf{E}_0 \cdot \mathbf{r} (r < a) \tag{86a}
\]

\[
\Phi(\mathbf{r}) = -\mathbf{E}_0 \cdot \mathbf{r} + \frac{\varepsilon - \varepsilon_d}{\varepsilon + 2\varepsilon_d} \frac{\mathbf{p} \cdot \mathbf{r}}{r^3} (r > a) \tag{86b}
\]

which is also valid for any direction of the electric field \( \mathbf{E}_0 \). So far, we see from equation (85) that the metal particle produces a static dipole field proportional to the applied electric field. Hence, by introducing the polarizability \( \alpha(\omega) \) via the relation \( \mathbf{p} = \varepsilon_0 \varepsilon_d \alpha(\omega) \mathbf{E}_0 \), equation (85) yields the important result

\[
\alpha(\omega) = 4\pi a^3 \frac{\varepsilon(\omega) - \varepsilon_d}{\varepsilon(\omega) + 2\varepsilon_d}.
\]

While the origin of the first resonance type was analyzed in section 2.5, in this section the concept of quasiguided waveguide modes is discussed. The basic dispersion properties of these modes is motivated by first discussing the properties of guided modes and subsequently making the transition to periodically perturbed waveguide slabs via the empty lattice approximation. Since this approach is based on highly idealized assumptions, it does not allow to accurately calculate the optical response of the structures introduced in later sections. However, it gives a basic understanding of the most important properties of quasiguided waveguide modes and their origin. This section is based on the discussions in [37–39, 47, 72–74].

Figure 14 shows a schematic drawing of a dielectric slab waveguide (light blue) on top of a substrate (grey). The superstrate above the waveguide is drawn as empty white space. The dark blue features indicate a periodic perturbation that is assumed to involve only nonmetallic elements. Furthermore, the perturbation is assumed to be so small that the influence on the effective refractive index of the slab waveguide is negligible.

Furthermore, as we will see in section 4, LSPRs also occur in less confined geometries, such as metal wires, if the light polarization is perpendicular to the wire, i.e. in the direction of the metal confinement. In analogy to the case of spheres, a plasmon resonance in a gold wire also red-shifts for increasing wire width.

2.6. Resonances in grating-waveguide structures

The Faraday rotation enhancement mechanisms utilized in the sections 4–6 are based on modifying magneto-optic thin films such that they provide both localized surface plasmon resonances, as well as quasiguided waveguide resonances. While the origin of the first resonance type was analyzed in section 2.5, in this section the concept of quasiguided waveguide modes is discussed. The basic dispersion properties of these modes is motivated by first discussing the properties of guided modes and subsequently making the transition to periodically perturbed waveguide slabs via the empty lattice approximation. Since this approach is based on highly idealized assumptions, it does not allow to accurately calculate the optical response of the structures introduced in later sections. However, it gives a basic understanding of the most important properties of quasiguided waveguide modes and their origin. This section is based on the discussions in [37–39, 47, 72–74].

Figure 14 shows a schematic drawing of a dielectric slab waveguide. It consists of a thin waveguide (WG) layer with thickness \( h \) and permittivity \( \varepsilon_{WG} \) on top of a substrate with permittivity \( \varepsilon_{\text{sub}} \). The superstrate, i.e. the half-space above the waveguide layer (usually air), is assumed to possess the dielectric function \( \varepsilon_{\text{super}} \). In order to allow light to be guided inside the center slab, the dielectric functions have to fulfill the condition \( \varepsilon_{WG} > \varepsilon_{\text{super}}, \varepsilon_{\text{sub}} \). The entire structure is assumed to be extended infinitely in \( y \)-direction. The dark blue markings indicate a periodic perturbation of the waveguide layer with periodicity \( \mathbf{p} \). For example, this could be a corrugated upper surface of the waveguide film or a dielectric grating with a small extension in \( z \)-direction. In later sections, it is shown that in the
case of a metallic grating, additional plasmonic resonances are introduced to the system. However, in this section only the case of purely dielectric perturbations is considered.

At first, let us consider the situation without any periodic corrugation. In this case the waveguide consists of three flat and homogenous layers. The corresponding optical eigen-modes are obtained by solving the Helmholtz wave equations

\[ \Delta E(r) + k_0^2 E(r) = 0 \]
\[ \Delta H(r) + k_0^2 H(r) = 0. \]

A comparison with (18) shows that the substitution \( k_0 = \omega/c_0 \) was used. It can be shown that for the considered geometry there are two types of solutions [47]: transverse electric (TE) modes (with \( E = E_x e_y \), \( H = H_y e_x + H_z e_z \)) and transverse magnetic (TM) modes (with \( H = H_y e_x \), \( E = E_x e_y + E_z e_z \)). For the TE modes the following ansatz can be made:

\[ E_y(x, z) = Ae^{-k_y z} e^{i(k_x x - \omega t)} \] (superstrate) \hfill (93)
\[ E_y(x, z) = (Be^{ik_x z} + Ce^{-ik_x z}) e^{i(k_x x - \omega t)} \] (WG slab) \hfill (94)
\[ E_y(x, z) = De^{-k_y z} e^{i(k_x x - \omega t)} \] (substrate) \hfill (95)

The ansatz for \( H_y \) in the case of TM modes (not shown here) is very similar. By applying the correct boundary conditions for the material interfaces, the following dispersion relation for TE modes is obtained [47, 74]:

\[
\begin{align*}
    h \sqrt{k^2_0 \varepsilon_{WG} - k^2_x} &= \arctan \left[ \frac{k^2_0 (\varepsilon_{WG} - \varepsilon_{super})}{k^2_0 \varepsilon_{WG} - k^2_x} - 1 \right] \\
    &+ \arctan \left[ \frac{k^2_0 (\varepsilon_{WG} - \varepsilon_{sub})}{k^2_0 \varepsilon_{WG} - k^2_x} - 1 \right] + m\pi.
\end{align*}
\]

For TM modes the dispersion relation is given by

\[
\begin{align*}
    h \sqrt{k^2_0 \varepsilon_{WG} - k^2_x} &= \arctan \left[ \frac{\varepsilon_{WG}}{\varepsilon_{super}} \frac{k^2_0 (\varepsilon_{WG} - \varepsilon_{super})}{k^2_0 \varepsilon_{WG} - k^2_x} - 1 \right] \\
    &+ \arctan \left[ \frac{\varepsilon_{WG}}{\varepsilon_{sub}} \frac{k^2_0 (\varepsilon_{WG} - \varepsilon_{sub})}{k^2_0 \varepsilon_{WG} - k^2_x} - 1 \right] + m\pi.
\end{align*}
\]

The equations (96) and (97) are transcendental equations, which connect the frequency of the light \( \omega = c_0 k_0 \) with the propagation constant \( k_x \). The integer \( (m = 0, 1, 2, 3,...) \) denotes the mode order. For \( \varepsilon_{super} \neq \varepsilon_{sub} \) the dispersion relations only possess solutions for photon energies \( E = c_0 k_0 \) above the cut-off energy \( E_{cut} \). These cut-off energies are

\[ E_{cut,TE} = \frac{\hbar c_0}{h \sqrt{\varepsilon_{WG} - \varepsilon_{sub}}} \times \arctan \left( \frac{\varepsilon_{sub} - \varepsilon_{super}}{\varepsilon_{WG} - \varepsilon_{sub}} \right) + m\pi \] (98)

for TE modes and

\[ E_{cut,TM} = \frac{\hbar c_0}{h \sqrt{\varepsilon_{WG} - \varepsilon_{sub}}} \times \left[ \arctan \left( \frac{\varepsilon_{sub} - \varepsilon_{super}}{\varepsilon_{WG} - \varepsilon_{sub}} \right) + m\pi \right] \] (99)

for TM modes. Figure 15 displays an exemplary dispersion plot obtained by numerically solving the transcendental equations (96) and (97) for \( m = 0 \). The assumed structure parameters are \( \varepsilon_{WG} = 3.61 \), \( \varepsilon_{super} = 1 \), and \( \varepsilon_{sub} = 2.1 \), as well as \( h = 140 \text{nm} \). The energies of the TE and TM waveguide modes are plotted with blue and orange solid lines, respectively. The corresponding cut-off energies are marked with dotted lines. The light cones of air, waveguide material, and substrate are drawn as black lines. From the plot it can be seen that the modes are always above the light cone of the waveguide material. This means that for a given light frequency, the effective wavelength inside the waveguide slab is always longer than for an infinitely thick waveguide slab (i.e. bulk).

In order to realize the excitation of guided modes by incident light, the following three requirements have to be met:

1. The polarization of the incoming light has to match the polarization of the waveguide modes. This means that TE\(_0\) and TM\(_0\) waveguide modes can only be excited by s-polarized and p-polarized light respectively.

2. Energy conservation requires that the incoming light and the guided light have to have the same frequency \( \omega \).

3. Momentum conservation requires that the component of the incoming wave vector (see figure 14) matches the wavenumber inside the slab, i.e.

\[ k_x = k_x^{\parallel}. \] (100)

The latter two conditions implicate that guided modes cannot be excited by incident plane waves: looking at figure 15 reveals that for a given light frequency (energy), the propagation
constant \( k_x \) is always larger than the wavenumber of the light cones of air and the substrate. This means that even for incident light with \( k_x^l \rightarrow |k_x| \), the equation (100) cannot be fulfilled.

One way to overcome the problem of simultaneous energy and momentum conservation is by utilizing prism couplers \([75, 76]\). This technique employs a prism with a refractive index larger than that of the waveguide slab. This prism is brought in contact with the waveguide slab and light is sent through the prism such that it is reflected at the contact interface. Due to photon tunneling, an evanescent wave is then leaking into the waveguide slab. The \( x \) component of this evanescent wave can be matched with \( k_x \) by adjusting the incident angle into the prism.

Another way of coupling the waveguide modes to external plane waves can be realized by introducing periodic perturbations of the waveguide surface, e.g. by attaching a grating on top of the waveguide \([47, 73]\). In figure 14 such a perturbation is indicated by the dark blue features. If this perturbation is small and the change of the effective refractive index of the waveguide slab is negligible, the modal dispersion can be assumed to be the same as in the non-disturbed case, with the only difference that the range of \( k_x \) can now be reduced to the first Brillouin zone \((-\pi/p \leq k_x \leq \pi/p)\) by treating every wavenumber \( k_x \) to be equivalent to \( k_x \pm 2\pi/p \), with \((l = 0, 1, 2, 3...)\) \([47]\). As a result, the condition (100) relaxes to

\[
k_x = k_m^l \pm \frac{2\pi}{p}, \quad (l = 0, 1, 2, 3..).
\]

This means that the dispersion line is folded back at the borders of the first Brillouin zone \((k_x = \pm \pi/p)\). This description is called empty lattice approximation \([72]\). Exemplary results of this approximation are displayed in figure 16. The two plots show the modal dispersion of the same slab waveguide as discussed in figure 15 but with an additional periodic perturbation with \( p = 300\) nm and \( p = 550\) nm. For better readability, only the TE\(_0\) waveguide mode is plotted. The most notable feature of these plots is that as a consequence of the folding of the dispersion line, the TE modes now reach regions above the air and substrate light cones. In this region the corresponding TE waveguide modes no longer possess an infinite lifetime and can couple to incoming and outgoing plane waves, i.e. the modes become quasi-guided. Of special relevance for section 4 are the points where the dispersion lines intersect with the line of \( k_x = 0 \). These points correspond to the case of normal incidence \((k_m^l = 0\) and \( l = \pm 1, \pm 2, ...\)). For example, in the case of \( p = 300\) nm the first mode accessible for normal incidence has an energy of \( E_1 = 2.5\) eV. On the other hand, in the case of \( p = 550\) nm the first intersection with the energy axis already happens at lower energies such that also the second intersection at \( E_2 \) lies within the plotting range. At this point we note the important result that a larger grating period leads to a lower resonance frequency.

Both at the center \((k_x = 0)\) and at the edge of the first Brillouin zone \((k_x = \pi/p)\) the modes are twofold degenerated due to the line folding. At these points the periodic structure supports two modes with the same energy, but with different symmetry of the electric field distribution with respect to the unit cell of the periodic waveguide. In the simplest case, one of the two modes shows a sinusoidal behavior with respect to the \( x \) direction, whereas the other one shows a cosinusoidal behavior. For normal incidence and systems with high symmetry, usually one of the two eigenmodes is dark, i.e. it possesses a line width of zero, and only the other mode can be excited efficiently. For an increasing incident angle, the line width of the dark mode increases and the mode becomes bright (see also section 4.3). Furthermore, when the periodic corrugation of the waveguide becomes stronger, the degeneracy of the eigenmodes at the center and the edges of the first Brillouin zone is lifted. This is in analogy to the case of electronic Bloch wavefunctions in solids \([77]\) and is indicated by the magnified crop next to \( E_1 \) in the right plot of figure 16.

The simple concept of the empty lattice approximation gives a good flavor of the nature of quasi-guided modes in corrugated slab waveguides. However, in most real world grating-waveguide systems, the periodic corrugation cannot be treated as a small perturbation anymore. This is also the case for the structures investigated in sections 4–6. Hence, full numerical simulations are required to solve Maxwell’s equations in order to accurately predict the modal dispersion and the optical response of such systems. In section 4 we will see that especially the introduction of metallic wire gratings dramatically influences the waveguide dispersion. As the individual grating wires provide an additional localized surface plasmon resonance, which can couple to the quasi-guided waveguide modes, a new hybrid excitation emerges: the so-called waveguide-plasmon-polariton.

3. State of the field

Over the last years the combination of plasmonic and dielectric nanostructures with magneto-optics resulted in a multitude of different nanoscopic systems with intriguing functionalities ranging from enhanced nonreciprocal optical effects to...
magnetically controlled optical devices. In this section we provide an overview of magnetoplasmonics and nonreciprocal photonics by systematically grouping the major contributions to the field. To complement our discussion, we also want to make the reader aware of several excellent reviews that focus on different subfields in the realm of plasmonics and magneto-optics [5, 7, 8, 78–85]. Parts of this section have been adapted with permission from the article [40].

3.1. Nonreciprocal photonics

The high demand for down-sized optical isolators stimulated a lot of research, which can be divided into two fields: on one hand people studied various ways to enhance the effective Verdet constant of conventional materials. Here, the utilization of plasmonic nanostructures has proven to be a very powerful method as it will be shown in the subsequent sections. On the other hand, there have been realized several nonreciprocal optical systems which do not directly rely on Faraday rotation. These approaches are quite diverse but can be classified with respect to how Lorentz reciprocity is broken.

One way of breaking Lorentz reciprocity is through nonlinear optical effects [7]. This approach is appealing as it allows for passive devices without any external magnetic bias. In particular, for future on-chip environments this aspect is very desirable. For example, much attention was received by the work of Fan et al. in 2012. They realized a CMOS-compatible all-silicon optical isolator based on ring resonators with a size of 5 μm [14]. The working principle of their two-port device is depicted in figure 17. It relies on the following mechanism: the two ports (silicon waveguides) are coupled via evanescent fields to a nonlinear silicon ring resonator acting as add-drop filter (labeled ADF). Port I is coupled more weakly to the ring resonator than port II. Hence, the amount of energy stored in the ring resonator is different for forward and backward light propagation. As the ring resonator exhibits a strong Kerr nonlinearity, the resonance frequency and thus the frequency of highest transmittance of the system changes depending on the propagation direction. By suitably adding a second ring resonator acting as a notch filter (labeled NF) the overall system exhibits asymmetric transmission with a transmission ratio of 20 dB. The drawback of this approach is the rather large insertion losses in the order of 30–40 dB. This is also a very common problem among other nonlinear isolators with high transmission ratios [86–88]. On the other hand, nonlinear isolators with low insertion losses often come with rather low transmission ratios [89–91]. However, in a recent theoretical study by Mahmoud et al. it has been suggested that nonlinear optical isolation could reach practical performance data by utilizing metamaterials [92]. That being said, it has to be stressed that for all passive nonlinear isolators the difference of forward and backward transmittance depends on the light intensity and vanishes for small intensities. This property disqualifies nonlinear isolators for many applications. Furthermore, in a recent study by Shi et al. it was demonstrated analytically that any optical isolator based on Kerr nonlinearities (as for example presented in the works [14, 87, 93–97]) cannot block a class of low intensity noise in the backward direction when a forward signal is transmitted [7]. They also suspect that this limitation could hold for a much broader range of nonlinear optical isolators.

Another way to break Lorentz reciprocity is spatiotemporal modulation [8]. For example, by electrically [98–105] or mechanically [106–110] modulating the local optical properties of electromagnetic wave carrying structures nonreciprocal operation has been proposed theoretically [98–100, 109–111] and demonstrated experimentally [101–108]. However, while time modulated systems reached practical relevance within the microwave regime, optical realizations are often only theoretical or proof-of-concept realizations [8]. This is because such systems usually come with dramatic losses and it is very difficult to obtain robust modulation with high strength and speed simultaneously [8].

As discussed in the beginning, magnetic fields are the most common way to break Lorentz reciprocity. Besides isolators relying on the Faraday effect there have been demonstrations of nanoplasmonic isolators based on plasmonic crystals [112] as well as magneto-optic waveguides and ring resonators [13, 113–115]. Among these, the most prominent work is the one by Bi et al. [13], where a monolithically integrated isolator with 19.5 dB isolation ratio at 1550 nm was experimentally demonstrated. The working principle of this system is based on the effect that the resonance frequency of a silicon ring resonator acting as notch filter depends on light circulation direction. However, as in the case of similar approaches [13, 113–115] there is a dramatic trade-off between isolation ratio and insertion loss. In the case of Bi et al. the insertion loss is in the order of 20 dB, attenuating the light intensity by a factor of 100.

Furthermore, Davoyan et al. [116, 117] theoretically proposed a 2D nanoplasmonic circulator with three ports. Such a nonreciprocal device routes light coming from one port into a specific port depending on the propagation direction. The core element connecting the three junctions consists of plasmonic nanorods embedded inside a magneto-optic bismuth-iron-garnet host. While the fabrication of such a structure might be challenging, Davoyan et al. demonstrated numerically that under ideal conditions the transmission through one of the
output ports can reach up to 63%, while the transmission of the other (isolated) port is well below 3% [116].

Recently, Scheucher et al demonstrated that a circulator can also be realized with a whispering gallery resonator and an attached single rubidium atom [118]. The researchers demonstrated that depending on the quantum state of the atom the optical throughput of two tapered optical fibers attached to the resonator can be routed. The isolation ratios between pairs of the four ports have been demonstrated to be in the order of 10 dB with an insertion loss of around 1.4 dB. Such quantum systems involving so-called chiral coupling have been studied extensively over the last years [119]. Recently, there have been findings in 2D materials that utilize excitons in MoS2 for optical spin-orbit coupling and spin-momentum locking [120, 121]. These effects might lead also to compact nonreciprocal devices in the future.

An interesting approach towards thin film Faraday isolators is the ansatz by Christofi et al [122]. In their recent theoretical study, they utilize BIG nanodisks embedded in silica, which form high-index resonators for the telecom wavelength range. Through careful engineering, they are able produce a spectral overlap of the electric and magnetic dipole resonances (i.e. the fundamental and higher order Mie resonances). This way it is possible to achieve mostly forward scattering without reflection losses. Such a so-called Huygens metasurface fulfills the Kerker condition [123, 124]. In principle, by utilizing the analog to electromagnetically induced transparency, they can achieve a very high transmission above 90%. Depending on the quality factor of the resonance, Faraday rotations of up to 8° are possible for films of 260nm thickness at room temperature. To achieve the desired 45° rotation required for a Faraday isolator it might be necessary to stack multiple metasurfaces or to further optimize the quality factor. Furthermore, it would be interesting see how well experimental values match the theoretical predictions which neglected absorption losses.

At this point we should also note that in addition to all the above mentioned nonreciprocal systems there have also been studies on reciprocal systems with asymmetric light transmission [125]. Such systems mimic some behavior similar to optical isolators but with less restricted symmetry properties [48, 126–130]. However, we stress that optical isolators in the common sense need to break Lorentz reciprocity [5].

3.2. Magnetoplasmonic antennas

Now we come back to the concept of enhancing magneto-optic (MO) effects of conventional materials by means of plasmonic nanostructures. The most straightforward way to achieve this is to utilize magnetoplasmonic antennas, i.e. plasmonic nano-antennas which at least partially consist of MO materials. Here, the term antenna refers to any kind of metal nanostructure that provides a localized surface plasmon resonance (LSPR). Possible geometries range from colloidal metal nanoparticles [21] to more well defined systems as for example disks [32] or donuts [131]. It has been demonstrated both experimentally and theoretically that at the LSPR the MO response of magnetoplasmonic antennas is enhanced [30, 132]. The reason for this behavior is that at the plasmonic resonance not only the diagonal elements of the particle polarizability tensor are resonantly enhanced but also the off-diagonal elements [30, 132], which are the origin of MO activity (see section 2.2.2). For spherical MO particles this behavior can be described analytically via Mie theory [133, 134]. We note in passing that in addition to the resonant plasmonic enhancement there can also be non-resonant contributions to the MO enhancement, which are geometrically induced [135] but usually weaker than the resonant contributions.

While the first theoretical prediction of LSPR-enhanced MO response of plasmonic particles was made in 1987 [136] it was first observed experimentally over 10 years later in granular CoFe-HfO2 films [137] and in nickel nanowires embedded in alumina [138]. From there on the field of magnetoplasmonics started to emerge. Fabrication techniques such as hole mask lithography and electron beam lithography allowed to study the MO response of geometrically more well defined magnetoplasmonic antennas such as nickel disks [139–141]. As a result, design rules have been derived regarding the disk shape [142] and lattice parameters [33] in order to obtain a tailored and optimized MO response.

While ferromagnetic metals such as cobalt, nickel and iron exhibit very strong MO response, they are also extremely lossy (see section 2.4). This leads to rather broad plasmon resonances in nanoparticles and thus weak MO enhancement. Furthermore, the plasmonic features in the optical spectra are not very well defined. The cleanest plasmon resonances can be obtained from noble metal particles such as gold and silver antennas. Although such nanostructures exhibit rather weak MO activity they have proven to be very useful model systems for in-depth studies on the connections between LSPRs and enhanced MO response. For example, they have been utilized to confirm the accuracy of simple theoretical models for magnetoplasmonic antennas based on discrete dipole approximation [30, 143] and harmonic oscillators [144].

In order to combine high quality plasmon resonances of noble metals with strong MO activity, there have been several intriguing approaches based on hybrid systems. Already very early works on colloidal systems involving both a noble metal and a ferromagnet suggested that the noble metal contribution to the LSPR resonance can lead to a significant MO enhancement [21, 146]. However, as in the case of purely ferromagnetic antennas, by means of lithographic fabrication techniques more control over antenna geometry and composition can be realized. For example, González-Díaz et al demonstrated that sandwiches of Co/Au/Co disks fabricated by hole mask lithography exhibit a resonantly enhanced MO Kerr effect around the plasmonic resonance [147]. Furthermore, this enhancement can be tailored and optimized by choosing an appropriate thickness for the chromium and gold layers [31]. Most notably, Banthi et al [32] demonstrated that by including a dielectric layer inside a Au/Co/Au sandwich (see figure 18), the electric field distribution of the LSPR mode can be redistributed in such a way that an optimized tradeoff between light absorption and MO response can be achieved. This resulted in a Kerr rotation angle of over 0.3° associated with an extinction of below 0.1 [32].
Interestingly, it is not strictly required that the MO material is part of the resonant antenna itself. It is also possible to enhance the MO response of a continuous (non-resonant) metallic MO film by means of the near field of attached noble metal antennas [148–150]. Similarly, for dielectric MO films it has been demonstrated both theoretically [151] and experimentally [152, 153] that by incorporating noble metal antennas inside or on top of the surface of the MO film the Faraday effect can be enhanced significantly. Baryshev et al reported an enhancement of up to 10 times relative to a bare host [153]. However, such systems exhibit extremely low light transmittance in the order of 1% for a 100 nm thick structure. This limits the applicability for photonics applications. To overcome this problem, it was also suggested to compensate for the optical losses by introducing gain by additional pumping [154].

The combination of noble metal antennas with continuous MO films allowed to study several physically interesting coupling effects. For example Torrado et al [149] studied magneto-optical effects of localized plasmon resonances interacting with propagating ones resulting in strongly magnetic field dependent SPP dispersion (more discussion on this aspect is provided in section 3.3). Furthermore, Armelles et al [150] investigated the interplay between a continuous Au/Co multilayer and a chiral plasmonic oligomer leading to an overlay of chirality induced natural circular dichroism (NCD) and magnetic-field induced circular dichroism (MCD). Similarly, in a very recent study Zubritskaya et al [155] demonstrated a metasurface consisting of chiral arrangements of gold and nickel disks which provides magnetically tunable chiroptical response. In general magneto-optic and chiroptical effects can cross-couple and lead to so-called magneto-chiral dichroism (MChD), as it was demonstrated by Eslami et al for nickel nano-spirals [156]. This phenomenon describes the situation when the effective permittivity tensor of an optical system exhibits a term proportional to both the wavevector and the magnetic field [156]. While the occurrence of MChD is clearly very interesting its origin is not fully understood yet [157].

Although the antenna-enhanced MO response in the approaches above is very interesting from a fundamental standpoint, the application in miniaturized nonreciprocal devices, such as Faraday rotators is limited due to either high losses or still too weak MO effects. However, a recent study by Maccaferri et al brought up a novel application for magneto-plasmonic antennas, namely plasmonic sensing [145]. Conventional plasmonic sensors rely on tracing a shift of a plasmonic resonance of a nano-antenna resulting from a change of the surrounding refractive index [158] or due to gas exposure [69].

A typical figure of merit for such a sensing mechanism is the ratio of resonance shift for a given environmental change divided by the resonance linewidth in the absorbance spectrum [69]. This means that the resulting measurement accuracy is often limited by the broad linewidth of plasmonic resonances. To obtain significantly increased sensitivity Maccaferri et al [145] proposed to not monitor changes of the absorbance spectra but instead to leverage changes in the MO spectra. More precisely, they suggested to monitor the polarization ellipticity variation $\Delta \varepsilon$ of the light transmitted by an ensemble of Ni metal disks. An AFM image of such an ensemble is depicted in figure 19(a). Figure 19(b) shows a comparison of the extinction spectra (top panel) and the spectra of the quantity $1/|\Delta \varepsilon|$ (bottom panel) for different surrounding refractive indices. The blue, magenta, green and red curves correspond to air ($n=1$), water ($n=1.33$), 50% Vol. glycerol ($n=1.41$), and glycerol ($n=1.47$) respectively. Maccaferri et al reported that the sharp features in the $1/|\Delta \varepsilon|$ spectra indeed result in a real sensitivity advantage when compared to absorbance based measurements. As such, magnetoplasmonic antennas could form the basis of highly sensitive label-free biosensors.

### 3.3. Propagating surface-plasmon-polaritons in a magneto-optic environment

Besides localized surface plasmon resonances also propagating surface plasmon polaritons (SPP) can exhibit interesting behavior in an magneto-optically active environment. Studies of such systems can be divided into two groups: one on one hand it has been investigated how the dispersion behavior of SPPs can be controlled via magnetic fields. The second group of studies concentrated on systems with SPP-enhanced MO effects. In the following, we will give a brief overview of both groups.

SPP are propagating electromagnetic waves at metal-dielectric interfaces (see section 2.5.2). They are widely regarded as a key mechanism for future on-chip optical devices as they allow to guide light through deep sub-wavelength structures [160, 161]. To obtain such devices with high functionality it is important to have the ability to manipulate and switch the SPP propagation at high frequencies. Conventional
SPP modulation techniques relied on the refractive index tuning of an active dielectric layer attached to the metal film. However, these approaches either enabled only weak modulation or slow response times [162]. A much more effective way of controlling SPP waves in future photonic devices could be magneto-optic systems. The reason is that the wavevector $\beta_M$ of a SPP at the surface of a magneto-optically active metal can be modulated via the metal’s magnetization $M$. This can be expressed by the relation

$$\beta_M = \beta + \Delta \beta(M),$$

where $\beta$ is the SPP wavevector in equation (80) for zero magnetization of the metal and $\Delta \beta$ is the magneto-optically induced perturbation [81]. Depending on the direction of $M$ relative to the surface and to the wavevector of the SPP, $\Delta \beta$ is either linear or quadratic in $M$. A comprehensive overview of the different scenarios can be found in the work by Armelles and coworkers [81].

While purely ferromagnetic systems provide the strongest magnetic modulation of SPP waves, they also exhibit large ohmic losses. For that reason, similar to the case of magnetoplasmonic antennas, there has been efforts to combine long SPP propagation distances of noble metals with the strong MO response of ferromagnets. Most notably, in 2010 Temnov and coworkers [159] reported magnetically modulated SPP waves in Au/Co/Au films as illustrated in figure 20. They achieved a relative modulation of the SPP wavevector in the order of $10^{-4}$ for relatively low applied magnetic field strengths of 20 mT enabling potentially high modulation frequencies in the gigahertz regime [162]. In later follow-up work it was demonstrated that the sensitivity of the SPP dispersion can be further enhanced by an additional dielectric layer [163] and strongly modified by adding resonant plasmonic antennas [149].

Instead of controlling SPP waves magneto-optically, the opposite mechanism, namely utilizing SPP waves to influence MO effects, is also feasible. As already explained in section 2.5.2 the excitation of SPP waves by incident plane waves usually requires special momentum matching techniques. For the first experimental investigations of SPP enhanced MO effects prism couplers were utilized to launch SPP waves on purely ferromagnetic films [164] and later on Co/Au/Co multilayer films [165]. In these works it has been demonstrated that the SPP excitation results in enhanced MO Kerr effect in reflection geometry [164, 165]. Stimulated by these results, in the following years numerous studies have been conducted on optimizing different variations of such geometries [166–170].

It turned out that especially in the case when the magnetization of the film is in plane and perpendicular to the SPP wave vector, the increased MO response can be attributed to the magnetic modulation of the SPP wave vector discussed in the last paragraph [81].

In contrast to continuous films, periodically corrugated systems allow the excitation of SPP resonances and enhanced MO response without a prism coupler [171–177]. This aspect makes such systems much more suitable for the integration in an ultra-compact environment. In this category of magnetoplasmonic systems, especially plasmonic crystals employing the concept of extraordinary transmission [160] added strong momentum to the whole field. For example, in

Figure 19. (a) AFM profile of Ni magnetoplasmonic nanoantennas on glass. (b) Extinction spectra (top panel) and $1/|\Delta \epsilon|$ spectra (bottom panel) for different values of the surrounding refractive index. Adapted with permission from Macmillan Publishers Ltd: Nature Communications [145], Copyright (2015).
2007 a lot attention was received by the theoretical work by Belotelov and coworkers [63]. They demonstrated by means of rigorous coupled-wave analysis that attaching a 2D gold fishnet structure on top of a dielectric MO active BIG thin film can result in both large Faraday rotation (up to $0.78^\circ$ for a thickness of less than 200 nm) and also high light transmission of 35% in the near infrared. Responsible for these exceptional performance data is the interaction of quasi-guided waveguide modes (see section 2.6) inside the BIG film with SPP waves in the plasmonic grating [63]. Subsequently, similar theoretical studies were performed on enhanced MO Kerr effect in 1D systems [178, 179]. While so far the particular two-dimensional system from 2007 has not been realized experimentally, in 2011 Belotelov and coworkers demonstrated the experimental realization [62] of the one-dimensional approach. A sketch of the utilized geometry is depicted in figure 21(a). They investigated the enhancement of the TMOKE signal of a dielectric BIG film when a 1D corrugated gold film is deposited on top. As most ferromagnets exhibit strong optical losses TMOKE is usually characterized in reflection geometry (see section 2.2.4). However the high transparency of BIG allowed them to measure in transmission geometry where the TMOKE parameter $\delta_T$ is defined as relative difference in transmittance for opposite magnetization directions:

$$\delta_T = \left| \frac{T(M) - T(-M)}{T(0)} \right|.$$  (103)

Belotelov et al estimated the TMOKE signal of the bare BIG film to be $\delta_T \approx 10^{-5}$ and found that by covering the film with

![Figure 20. Illustration of the SPP modulation technique by Temnov et al. A slit-groove pair is milled in a Au/Co/Au multilayer film using a focused ion beam. Surface plasmons are launched by the groove, propagate towards the slit, and interfere with the directly transmitted light to produce a periodic interference pattern along the slit axes. The oscillating magnetic field of an electromagnet is used to periodically switch the magnetization in the thin cobalt layer and thus modify the wave vector of surface plasmons. An imaging nano-optical set-up is used to record the magneto-plasmonic modulation signal. Adapted with permission from Macmillan Publishers Ltd: Nature Photonics [159], Copyright (2010).](image)

![Figure 21. (a) Sketch of the magnetoplasmonic crystal for enhanced TMOKE. (b) Measured transmittance and TMOKE parameter $\delta_T$ for an incident angle of 15$^\circ$. Adapted with permission from Macmillan Publishers Ltd: Nature Nanotechnology [62], Copyright (2011).](image)
a continuous gold layer the TMOKE signal is increased up to \( \delta_T \approx 10^{-3} \) through the excitation of SPP waves. Most interestingly, their measurement results showed (see figure 21(b)) that by introducing 1D slits into the gold film extraordinary optical transmission can be realized which further boosts the TMOKE signal up to \( \delta_T = 1.5 \times 10^{-2} \) for an incident angle of 15°. Moreover, in 2013 it was demonstrated for a similar geometry that a longitudinal magnetic field can increase the transmittance from 0.04 in the non-magnetized case by up to 24% [180].

These results raised the expectations for future plasmonically driven MO devices. However, being a surface effect, even the plasmonically enhanced TMOKE is still too weak for the most demanding magneto-optical devices such as isolators. As a result, there have been several efforts to enhance the potentially strongest MO effect, namely the Faraday effect [36, 40–42]. Although the investigated geometries in these works might seem quite similar to the ones utilized in the works discussed above, they rely on localized surface plasmon resonances rather than on propagating SPP waves. Hence, they are discussed separately in the following section 3.4. Furthermore, the sections 4–6 provide in-depth discussions of some recent studies that led to the strongest Faraday rotation in thin film geometries so far.

3.4. Waveguide-plasmon-polaritons

Through the discussions in the preceding sections, we have identified three main ingredients to realize a good trade-off between strong MO response and low light absorption in magnetoplasmonic systems: the first one is to utilize separate components responsible for strong MO activity and for pronounced plasmonic resonances [31]. The second ingredient is to decrease the losses by concentrating light inside a transparent dielectric material [32]. The third aspect is to also leverage waveguide mode excitations inside the MO material [62, 63].

All these three aspects are employed in hybrid magnetoplasmonic waveguides as introduced by Chin et al in 2013 [36]. Such systems consist of a dielectric MO waveguide with a thickness in the order of 150 nm and an attached noble metal grating on top (see figure 22). Unlike in the work by Belotelov et al [62, 63, 180] the noble metal grating is much less dense and does not support any propagating SPP waves. Instead, the gold grating has the following two main functions: Firstly, it acts as a waveguide coupler to enable the excitation of quasi-guided waveguide modes inside the MO film for normal incidence. Secondly, it provides a localized surface plasmon resonance that couples strongly to the TM-polarized high-Q waveguide mode inside the MO film leading to the formation of a so-called waveguide-plasmon-polariton (WPP). This quasi-particle was first studied in 2003 in a non-magneto-optic context by Christ and coworkers [37, 38]. However, Chin et al discovered that the dispersion properties of a WPP also facilitate strong enhancement of Faraday rotation [36]. In a simple picture this can be be understood in the following way: the high oscillator strength of the plasmonic wires enables coupling normally incident light efficiently into the system where it resonates inside the transparent MO film and accumulates Faraday rotation proportional to the Q-factor of the system. In the first realization by Chin et al a combination of a 150 nm thick BIG thin film with a 70 nm thick gold grating was utilized. It was demonstrated both experimentally and through numerical simulations that such sub-wavelength thick structures exhibit giant Faraday rotation of up to 0.8° and simultaneously high transmittance of 36% in the near-infrared. Furthermore, in a follow-up study it has been demonstrated that such waveguide-plasmon-polariton systems also show an extremely large TMOKE signal in transmission geometry of up to \( \delta_T = 1.5 \times 10^{-3} \) associated with a transmittance of 45% [34]. While the TMOKE performance of such WPP systems is comparable to the geometry by Belotelov et al [62], the obtained Faraday rotation performance was setting a new benchmark at that time.

Starting from there, in the following years there has been a series of works resulting in even thinner Faraday rotators with one order of magnitude larger rotation and more functionality. As the performance of these nanoscopic structures is not far away from meeting the requirements of a Faraday isolator, the concept of hybrid magnetoplasmonic crystals could be a key to realize nonreciprocal photonic devices in an highly integrated environment. The first step in this direction will be discussed in section 4. There we will elaborate on how the hybrid magneto-plasmonic approach by Chin et al has been employed and advanced [40] to 220 nm thick devices, which, at low temperatures, show five times greater polarization rotation than in previously reported experiments. In contrast to BIG, the utilized waveguide material EuSe provides a stronger magneto-optical response and also allows much simpler sample fabrication. This aspect is especially crucial.
as it opens the way for creating a class of more sophisticated 2D and 3D magnetoplasmonic metamaterials in the future. Furthermore, we discuss how the dispersion properties of such structures can be exploited to freely tailor the working wavelength within the transparency window of the magneto-optical material. In addition, the aspect of active magnetic polarization tuning will be examined.

While the experimental realization and numerical simulation of hybrid magnetoplasmonic systems received considerable attention, until recently, an analytical theoretical description has been missing. In section 5, a simple coupled oscillator model is presented, that reveals the underlying physics inside hybrid magnetoplasmonic systems and yields analytical expressions for the resonantly enhanced magneto-optical response [41]. The Lorentz nonreciprocity of the oscillator model is intrinsically incorporated via the Lorentz force, which is proportional to $\mathbf{v} \times \mathbf{B}$. Moreover, the predictions of the model are in good agreement with rigorous numerical solutions of Maxwell's equations for typical sample geometries. The demonstrated ansatz is transferable to other complex and hybrid nanooptical systems and could significantly facilitate device design.

Usually, the maximal MO response of WPP systems is limited by the low $Q$ factor of the plasmon resonances of the grating. This limitation is lifted elegantly by a novel approach [42] presented in section 6, which is based on a classical optical analog of electromagnetically induced absorption (EIA) [181, 182]. Here, a strongly damped plasmon oscillation is weakly coupled to a narrow linewidth waveguide resonance with a phase delay, leading to constructive interference. By tuning this coupling carefully, a high-$Q$ absorptive hybrid mode is realized, which can be used to resonantly amplify the Faraday rotation response. Furthermore, the EIA mechanism allows to utilize the large oscillator strength of the plasmonic resonance, leading to an efficient coupling of the incident light into the structure without reducing the effective $Q$ factor due to the broad plasmonic resonance as was the case in previous approaches [34, 36, 40]. Although less than 200 nm thick, at low temperatures, the novel EuS-Au structure design exhibits Faraday rotation of 14°. As this is only a factor of three away from the Faraday isolation requirement, the demonstrated factor of three away from the Faraday isolation requirement, the demonstrated concept might lead to important, highly integrated, nonreciprocal, photonic devices for light modulation, optical isolation, and optical magnetic field sensing. Moreover, the simple fabrication of EuSe nanostructures by physical vapor deposition opens the way for many interesting magneto-plasmonic systems and 3D magneto-optical metamaterials. This section has been adapted with permission from the dissertation [43] and the article [40].

4. Giant Faraday effect in EuSe-Au structures

In this section, the realization of an ultra-thin plasmonic Faraday rotator for the visible wavelength regime will be demonstrated. The rotator is a magneto-plasmonic hybrid structure consisting of an EuSe slab and a 1D plasmonic gold grating. At low temperatures, EuSe possesses a large Verdet constant and exhibits Faraday rotation, which does not saturate over a regime of several Tesla. By combining these properties with plasmonic Faraday rotation enhancement, as introduced by Chin et al [36], giant Faraday rotation of up to 4.2° for a film thickness of only 220 nm is achieved. This magneto-optic response is five times stronger than in previously reported experiments. Furthermore, by varying the magnetic field from -5 to +5 T, the polarization of the transmitted light can be continuously tuned over a range of 8.4°. Through experiments and simulations, it will be demonstrated for the first time that the unique dispersion properties of such a Faraday rotator allow to tailor its working wavelength to arbitrary spectral positions within the transparency window of the magneto-optical slab. The demonstrated concept might lead to important, highly integrated, nonreciprocal, photonic devices for light modulation, optical isolation, and optical magnetic field sensing. Moreover, the simple fabrication of EuSe nanostructures by physical vapor deposition opens the way for many interesting magneto-plasmonic systems and 3D magneto-optical metamaterials. This section has been adapted with permission from the dissertation [43] and the article [40].

4.1. The waveguide-plasmon-polaritonic approach

From the discussion of the Faraday effect in section 2.2.3, we have seen that, in first approximation, the Faraday rotation angle scales linearly with the applied magnetic field, the material specific Verdet constant and the material thickness. Hence, as the rotation capabilities of a magneto-optic material is usually limited by a small Verdet constant, a thin-film rotator structure might seem contradictory. However, a strong Faraday effect and a low material thickness can be combined by making the rotator structure resonant. As it was also discussed in section 2.2.3, the Faraday effect is non-reciprocal, which allows light to accumulate rotation of the same sign and magnitude for both forward and backward propagation. This behavior implies that when light propagates through a medium and is reflected in the backward direction, the accumulated Faraday rotation is twice the rotation of only one pass. Moreover, the rotation can be enhanced even further by additional round-trips through the medium. This principle is utilized in the magneto-plasmonic structure schematically depicted in figure 23(a). The structure consists of an EuSe slab waveguide and a gold wire grating on top. Both the magnetic field and the wave vector of the incident light are assumed be perpendicular to the EuSe film. In the following, the incident polarization parallel and perpendicular to the wires is referred to as TE and TM, respectively. The gold nanowire grating has two functions. First, it acts as a waveguide coupler and allows normally incident light to couple into the slab and excite a waveguide mode (see section 2.6). The wave vectors of these modes possess a significant component in the $z$-direction. In ray approximation, this scenario can be understood as light bouncing back and forth inside the magneto-optical film and accumulating the rotations of multiple round-trips. When light is eventually coupled out of the film, due to a finite lifetime, the effective Faraday rotation is significantly larger than that for a bare film.

Many applications require wavelength-specific device operation. The presented structure design provides the freedom to tune the working wavelength (i.e. the spectral region with largest rotation enhancement) to arbitrary positions.
within the transparency window of the magneto-optical film. To explain the principle behind the wavelength tunability, we must first consider the connection between Faraday rotation enhancement and the dispersion properties of the hybrid structure. The largest Faraday rotation enhancement occurs at wavelengths at which the structure supports both a TM-polarized and a TE-polarized eigenmode (see section 2.6). In that case, the TM-to-TE conversion (i.e. polarization rotation) can occur most efficiently. This behavior will also be described analytically in section 5. Strictly speaking, the modes excited by TE- and TM-polarized incident light are themselves not purely (but mainly) TE- and TM-polarized due to the anisotropic permittivity tensor of the magneto-optical material. However, in the interest of readability, these modes are referred to as TE and TM modes, respectively. To overlap the TE and TM modes in $k$-space as well as at their energetic position, the second function of the metallic grating comes into play. In TM polarization, the gold wires provide a localized particle plasmon resonance that hybridizes with the TM waveguide mode of the magneto-optical slab and forms a waveguide-plasmon-polariton (WPP) [38]. For the resulting TM-polarized WPP, there will always exist a grating period such that its dispersion curve intersects with a TE waveguide dispersion curve at the same wavelength. This mechanism is illustrated in figure 23(b), which schematically displays the dispersion behavior of the magneto-plasmonic hybrid structure. The lines in the diagram trace the resonance frequencies of the TE and TM modes for different grating periods. The solid lines correspond to the case when the grating coupler is made of a dielectric material and thus does not support any plasmonic resonance. As it was discussed in section 2.6, in the empty-lattice approximation, the waveguide resonance wavelength increases with larger grating periods. Since the exciting light wave is assumed to be impinging normally, the solid lines in the dispersion diagram can be associated with the energy $E_1$ in figure 16. The dispersion for a metallic waveguide coupler is depicted as dotted lines. In the dielectric case, the TE and TM modes never overlap except for the zero grating period or inclined incidence [37], whereas they overlap in the metallic case owing to the formation of the WPP. As a result of the coupling of the localized particle plasmon to the TM waveguide mode, the dispersion curve of the TM mode is bent over the TE waveguide-mode dispersion curve and an intersection is created [37, 38]. The position of this intersection directly depends on the wavelength of the localized particle plasmon in the grating wires. By increasing (decreasing) the wire width of the grating, both the plasmonic resonance and intersection point of the WPP and TE mode shift toward longer (shorter) wavelengths and larger (smaller) periods. By means of this feature, the working wavelength of the system can be tuned via the gold wire width and grating period.

In the experimental realization of the hybrid-plasmonic structure, the waveguide was made of EuSe. Already in the 1960s, in the context of research on computer memories, the group of Europium chalcogenides received attention for their exceptional magnetic properties and strong magneto-optical response at low temperatures [59, 183, 184]. In particular the compound EuSe exhibits extremely large Faraday rotation angles on the order of 1° per Tesla and micrometer thickness in the visible wavelength range at temperatures of 30 K. EuSe also possesses a high saturation magnetic flux density of 2–5 T, depending on the wavelength [183–185]. Furthermore, the fabrication of EuSe thin films is very simple and can be carried out by physical vapor deposition [186] (PVD). This feature is a significant advantage over BIG, which is widely utilized both in magneto-optical devices [187–190] and concept studies [14, 36, 162]. BIG films are typically fabricated by pulsed laser deposition [60] (PLD) followed by high-temperature annealing, which is largely restricted to homogeneous films and does not allow for the direct incorporation of other materials, such as plasmonic nanostructures, into the film. With the flexibility of PVD, EuSe provides the possibility of fabricating more sophisticated potential future designs, including 3D geometries, where the magneto-optical and plasmonic elements are merged. The gold gratings attached to the EuSe films were fabricated by electron beam lithography [191].

To support the explanation of the Faraday rotation enhancement mechanism, the dispersion behavior of the demonstrated structures was simulated using the scattering matrix method [192]. The simulation results displayed in figure 24 confirm...
the formation of WPPs in the EuSe-Au hybrid structure for TM incidence and also show that the maximum Faraday rotation enhancement can be expected around the intersection of the TE waveguide mode and the TM-polarized WPP. The upper panels of figure 24 show the simulated absorbance spectra for a 150 nm thick EuSe film with a gold wire grating on top for varying the grating periods. The gold wires are assumed to be 70 nm thick and 70 nm wide. The black and white dashed lines trace the radiating TE- and TM-mode resonance frequencies of the EuSe hybrid structure and have been derived \textit{ab initio} from the scattering matrix [192]. The red dashed line indicates the Rayleigh anomaly depending on the grating period. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

In the numerical simulations, the gold has been described by tabulated data by Johnson and Christy [193]. The glass substrate was assumed to possess a constant relative permittivity of 2.13. The diagonal elements of the permittivity tensor of the EuSe film were modeled using experimental data gained from transmission measurements of different plasmonic gratings on top of an EuSe slab. The off-diagonal elements of the dielectric tensor were obtained from measurements of a blank EuSe film at 30 K and 5 T, which is the condition given in the performed experiments.

4.2. Experimental realization

The Faraday rotation measurements for the EuSe-Au hybrid structures were performed in a magnet cryostat at 30 K. The polarization rotation was measured with a rotating analyzer between the illuminated sample and a spectrometer. Because the presented structures are anisotropic, for rotation measurements it is crucial that the incident light be either purely TE- or TM-polarized. For other incident polarizations, undesired reciprocal rotation contributions occur owing to different transmittance for TE and TM polarization. Wavelength-dependent deviations from the pure TE or TM polarizations can occur due to the Faraday rotation in glass elements within the reach of the magnetic field of the magnet cryostat. For this reason, an intra-cryostat polarizer was placed directly in front of the sample to ensure that the unpolarized measurement light beam is only polarized just before it hits the sample.

4.2.1. Faraday rotation enhancement. The simulated Faraday rotation of the EuSe-Au structures, presented in section 4.1, was subsequently confirmed experimentally. Figure 25(a) displays the transmittance and Faraday rotation spectra for TM incidence for different grating periods. The utilized EuSe film is 150 nm thick. Both the width and thickness of the gold wires is 70 nm. The corresponding
Figure 26. Experimental demonstration of sweeping the Faraday rotation enhancement over a range of wavelengths. The graph is showing the Faraday rotation of a 150 nm thick EuSe slab and 70 nm thick periodic gold wires. From left to right, the curves correspond to a wire width $w$ of (70 nm, 80 nm, 90 nm) and a grating period $p$ of (360 nm, 390 nm, 420 nm). Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.

spectra from the simulations are plotted in figure 25(b). The black curves represent the case of a blank EuSe film without grating. The blue curves correspond to a 360 nm grating period and represent the case where the TE- and TM-mode dispersions have the greatest overlap. Hence, the largest Faraday rotation enhancement is achieved. At 662 nm, the structure exhibits a Faraday rotation of 4.2° at reasonably high transmittance of 30%. The other curves show that when the period is increased, the enhancement of the rotation decreases as the TE and TM modes move farther apart. As in the color-coded absorbance diagram in figure 24, the resonance features of the TE mode (negative rotation contribution) and TM mode (positive rotation contribution) are drifting away from each other with an increasing grating period while simultaneously flattening out. The simulated and experimental data correspond well, although there is a difference in the Faraday rotation base line plotted in black. For small wavelengths, in the simulation, this curve completely overlaps with the colored rotation spectra of the different gratings. However, in the measurement this is not completely the case. This discrepancy is caused by the limited temperature accuracy of the utilized cryostat system. In principle, this effect can be completely eliminated by using a cryostat that can maintain a temperature below 7 K, which is the Neél temperature of EuSe. Earlier measurements (not shown here) confirmed that in this temperature range, the Faraday rotation of EuSe is even larger and not temperature dependent.

4.2.2. Tunable working wavelength. As explained in section 4.1, the Faraday rotation of the waveguide material can be enhanced at selected wavelengths. The only requirement is that the material offers a sufficiently low absorption at the wavelength of interest to achieve a high-quality factor of the waveguide resonator. From the visible to the near infrared, EuSe is transparent at wavelengths larger than 550 nm [194]. Figure 26 shows the measured Faraday rotation spectrum of a 150 nm thick EuSe film at a temperature of 30 K and a magnetic flux density of 5 T. The black curve denotes the Faraday rotation for the case without plasmonic enhancement. By tailoring the plasmonic gratings such that the TE waveguide mode and TM WPP mode dispersions overlap at different wavelengths, the Faraday rotation can be enhanced selectively at these tailored wavelengths. As it was explained in section 4.1, for a given thickness of the magneto-optical slab, the TE-TM overlap can be achieved by tuning the wire width and period of the metal wires. The colored curves in figure 26 correspond to three different plasmonic gratings, which fulfill the TE-TM-matching condition. All gratings share a constant wire thickness of 70 nm. Both the wire width $w = (70$ nm, 80 nm, 90 nm) and the grating period $p = (360$ nm, 390 nm, 420 nm) are increasing from left to right. The resulting maximum absolute values of Faraday rotation occur at the wavelengths $\lambda = (662$ nm, 695 nm, 736 nm). For longer wavelengths, both the intrinsic Faraday rotation of the film and the enhanced rotation are decreasing. This finding is in agreement with the assumption that the enhanced Faraday rotation and the intrinsic Faraday rotation of a film are approximately proportional to each other for constant absorption, a constant waveguide coupling efficiency, and thus a constant quality factor.

4.2.3. Magnetically tunable polarization rotation. The magnetic field dependence of the Faraday rotation was measured to demonstrate the polarization tuning capability of the demonstrated EuSe-Au structures. Figure 27(a) shows the Faraday rotation of the structure with a 360 nm grating period, 70 nm wire thickness, and 70 nm wire width for TM incidence. The magnetic field was varied from -5 T to +5 T. The structure geometry yields largest Faraday rotation at 663 nm. For this wavelength, a polarization rotation tuning range from $-4.2^\circ$ to $+4.2^\circ$ is obtained. The tuning behavior is nearly linear in the magnetic field, as can be extracted from figure 27(b). The polarization rotation measurement for inverted magnetic fields also acts as a control experiment to reveal potential spurious non-magnetically induced contributions. Since the measured samples are anisotropic, such effects can occur when the incident polarization deviates from an exactly TE or TM polarized state, for instance, as a result of the Faraday rotation of the cryostat windows. The highly mirror symmetric behavior of the measured rotation spectra in figure 27(a) clearly shows that polarization errors of the incident light are well under control.

Figure 27. Magnetic tuning of the polarization rotation. (a) Faraday rotation of a 150 nm thick EuSe slab and 70 nm thick and 70 nm wide periodic gold wires for different applied magnetic fields. (b) Magnetic field dependence of the Faraday rotation at 663 nm. Adapted from [40] with permission. © 2015 Macmillan Publishers Limited, part of Springer Nature.
4.3. On the incident angle and film thickness

The simulations and measurements presented in the sections 4.1 and 4.2 were performed for normal incidence. Here, the case of tilted incidence will be discussed by means of numerical simulations. As in the previous sections, the hybrid structure is assumed to consist of a 150 nm thick EuSe slab with 70 nm thick and wide gold wires on top. Furthermore, also a magnetic field of 5 T and a temperature of 30 K are assumed.

The optical path length through a magneto-optical film can be increased by tilting the incident beam. However, the Faraday rotation is proportional to the component of the optical path in the direction of the magnetization of the material, which is perpendicular to the EuSe film. Therefore, a small tilting angle does not affect the polarization rotation in a bare film. However, a different behavior is exhibited by the presented magnetoplasmonic hybrid structures. Their WPP dispersion, and thus, also the Faraday rotation response critically depends on the incident angle. This is illustrated by the absorbance plots in figure 28 for both TM and TE polarized incident light, as well as for different angles of the wavevector in the plane perpendicular to the gold wires. It can be clearly seen that for an increasing tilting angle, there emerges a second resonance feature both for TM and TE incidence. This is in agreement with the analysis of non-magneto-optic WPP systems by Christ et al. [37, 38], showing that for normal incidence there are two TE and TM polarized waveguide modes with similar frequency, where one of them is symmetry-forbidden and thus possesses zero line-width. However, by deviating from normal incidence the dark waveguide modes become bright. Furthermore, with an increasing tilting angle, the two resonances move further away from each other. In the empty lattice picture, illustrated in figure 16 for the TE modes, this behavior can be understood in the following way: for normal incidence ($k_x = 0$) the two branches of the TE mode overlap at energy $E_1$. This degeneracy is lifted when the tilting angle is increased ($k_x > 0$).

Comparing the modal dispersion in figure 28 with the corresponding Faraday rotation plots in figure 29 reveals that, regardless of the more complex spectra for tilted incidence, the criterion for maximum Faraday rotation enhancement is still the same. It occurs where the TM WPP has the strongest overlap with the TE waveguide modes.

It is also interesting to investigate the influence of the thickness of the magneto-optical EuSe slab. Since the Faraday rotation of a bulk piece of material scales with its thickness, it might seem reasonable that a thicker EuSe film also leads to larger Faraday rotation. However, this is not generally the case, especially when the grating period and the wire dimensions are kept the same. To illustrate this, let us consider the dispersion diagram in figure 23(b): increasing the film thickness induces a redshift of the waveguide modes, whereas the plasmonic resonance remains at the same spectral position (in first approximation). As a result, the TE-TM overlap region shifts to smaller periods. The consequence of this behavior is illustrated in figure 30, which shows the simulated Faraday rotation..
rotation dispersion spectra for different film thicknesses and different periods. The gold wires are assumed to be 70 nm thick and wide. The simulations confirm that the Faraday rotation of the film alone (far away from the TE and TM modes) increases for thicker EuSe films. However, since the TE-TM-overlap region is slightly shifted toward smaller periods, the Faraday rotation decreases for the listed periods. If desired, in principle, the shift of the TE-TM overlap could be compensated by tuning the plasmonic resonance toward longer wavelengths, which can be achieved by increasing the wire width.

4.4. Conclusion

An actively tunable thin-film optical rotator with a variable working wavelength was demonstrated both in experiment and in simulation. This was achieved by combining a slab of EuSe, which possesses a large Verdet constant and high saturation magnetic flux density, with a gold nanowire grating. The wavelength range of largest Faraday rotation can be selected by choosing the correct combination of grating period and wire width. The magnitude of optical rotation can be magnetically tuned over a wide angular range. At 30 K for a 220 nm thick structure, a rotation tuning range of up to 8.4° was obtained. For the present structure, this range can presumably be doubled when cooling down to temperatures below the Néel temperature of EuSe, which is 7 K.

The demonstrated concept can be expected to have applications in highly integrated optics, demanding actively controlled optical modulation [195], magnetic field sensing [196, 197] and optical isolation [12–14, 16–18]. Furthermore, the presented structure geometry is suitable for large-area fabrication [198, 199], which makes it a promising design for non-reciprocal coatings of optical elements, such as lenses, with active external control at specific wavelengths. In addition, the direct attachment onto optical fiber ends [200–202] or onto laser diodes could yield devices with extremely small volumes for a highly integrated environment.

The working principle of the presented structures is not restricted to EuSe as a waveguide material. It can be directly transferred to other magneto-optical materials. The only requirement is that the magneto-optical material provides a transparency window near the wavelength region of interest and a sufficiently high surface quality to ensure a high Q-factor. These requirements can be met by commonly used room temperature magneto-optical materials, such as BIG [60], YIG [60] or TGG [203]. For the low-temperature regime, there are also other chalcogenides, such as EuS [183], EuTe [184], EuO [204], which have similar magneto-optical properties as EuSe. The temperature at which these materials show the largest Faraday rotation can possibly be increased by doping with Gd. For example, doping EuO with Gd can raise the Curie temperature from 69 K to 135 K [57, 205]. EuSe and EuS combine large magneto-optical response, high saturation magnetic flux density, and simple thin-film fabrication via PVD. Thus, they are promising materials for further magneto-plasmonic studies with structures that are not restricted to designs including strictly continuous magneto-optical films. For example, in section 6, EuS will be utilized for a less than 200 nm thick structure design, that relies on an EuS thin film with an incorporated gold grating and produces Faraday rotation of up to 14°, corresponding to a tuning range of over 25°.

5. Lorentz force model for magnetoplasmonics

It was shown recently that the Faraday rotation [36] and also the transverse magneto-optic Kerr effect [34] of a dielectric film can be enhanced considerably by attaching a resonant plasmonic grating. As it is demonstrated in section 4, by varying the grating and nanowire geometry, the maximal polarization rotation enhancement can be tuned to arbitrary spectral positions [40]. Such structures exhibit Faraday rotation of up to 4.2° for a thickness of 220 nm [40]. Hence, they are very relevant for future devices, such as thin-film Faraday rotators and isolators as their performance exceeds other approaches considerably.

While the experimental realization and numerical simulation of such systems received considerable attention, so far, there has not been an analytical theoretical description. In this section, a simple coupled oscillator model will be presented, that reveals the underlying physics inside hybrid magneto-plasmonic systems and yields analytical expressions for the resonantly enhanced magneto-optical response. The Lorentz nonreciprocity of the oscillator model is intrinsically incorporated via the Lorentz force, which is proportional to $\mathbf{v} \times \mathbf{B}$. Moreover, the predictions of the model are in good agreement with rigorous numerical solutions of Maxwell’s equations for typical sample geometries. The demonstrated ansatz is transferable to other complex and hybrid nano-optical systems and will significantly facilitate device design.
In section 5.1 the model will be introduced and applied to the EuSe-Au structures discussed in the previous section. Next, the model will be assessed by comparing its predictions to full numerical simulations based on Maxwell’s equations. In order to maintain good readability, some of the mathematical details of the model are presented separately in the section 5.2.

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### 5.1. Application to EuSe-Au hybrid structures

The geometry of the investigated magneto-optical system is depicted in figure 31(a). It consists of a dielectric magneto-optic (MO) thin film with an attached one-dimensional gold grating on top. The incident light is assumed to be linearly polarized and impinges on the sample along the z-direction. The polarization direction with electric field perpendicular (parallel) to the gold wires will be referred to as x- (y-) polarized. Figure 31(b) depicts the corresponding mechanical analog of the optical system, where each relevant optical excitation is represented by a mass suspended by a linear spring. The coupling between the different excitations is modeled by interconnecting springs. All masses are assumed to be charged and driven by the external light field. Due to its periodicity, the gold grating acts primarily as a waveguide coupler and allows for the far-field excitation of transverse electric (TE) and transverse magnetic (TM) polarized quasi-guided waveguide modes inside the MO film. In the absence of the magnetic field, the TE (TM) waveguide modes can only be excited by y- (x-) polarized incident light. Thus, the waveguide modes are modeled by one mass each, which is restricted to move only along the x- or y-direction.

The second purpose of the gold wires is to provide a localized plasmon resonance, which can be excited by x-polarized light. This plasmonic resonance is taken into account by an oscillator moving in x-direction (labeled P). Due to the field overlap, the plasmonic resonance is coupled to the TM waveguide mode [37, 38]. The dielectric response of the MO material itself is modeled by the red mass, which can move within the xy-plane and is subjected to a Lorentz force [50] in the xy-plane due to a static magnetic field B oriented along z-direction.

The oscillator system in figure 31(b) possesses five degrees of freedom. Its motion is described by five coupled second order linear differential equations. While these equations can be solved exactly, it is impossible to derive closed expressions for its eigenmode frequencies. To simplify the model and allow for the analytical calculation of the eigenmode frequencies, a series of proper approximations can be applied.

First of all, neglecting the dispersion effects by the MO material itself yields significant simplification. This is achieved by assuming the driving frequency to be far away from the MO oscillator resonance and the impact of the driving force on the MO oscillator (i.e. the associated coupling constant) to be small, resulting in a reduced system of three coupled second order equations. Furthermore, in a rotating wave approximation, which is valid when the driving frequency ω is close to the eigenfrequencies \( \Omega_j \), \( j = \text{TE}, \text{TM}, P \) of the individual oscillators, the second order equations are reduced to first order. The mathematical details of the model reduction are provided in section 5.2 and the limitations of the applied approximations are discussed later in this section.

The simplified oscillator scheme is depicted in figure 31(c). The three masses of the TE, TM and material oscillator are now merged into one waveguide oscillator. Assuming a time-harmonic oscillator displacement that is proportional to \( \exp(-i \omega t) \), the governing equations in the rotating wave approximation are given by the matrix equation

\[
(M_0 + \Delta M - i \omega) x = \eta R E_{||},
\]

where \( I \) is the 3 x 3 identity matrix, and \( \eta \) is a residue of the rotating wave approximation that is inversely proportional to \( \exp(-i \omega t) \). The equations of the rotating wave approximation are given by the matrix

\[
M_0 = \begin{pmatrix}
\omega_{\text{TM}} & 0 & -\kappa \\
-\kappa & \omega_p & 0 \\
0 & 0 & \omega_{\text{TE}}
\end{pmatrix},
\]

Furthermore, \( M_0 \) accounts for the coupling of the TM waveguide mode and the plasmon [37], with

\[
M_0 = \begin{pmatrix}
\omega_{\text{TM}} & -\kappa & 0 \\
-\kappa & \omega_p & 0 \\
0 & 0 & \omega_{\text{TE}}
\end{pmatrix}.
\]

The corresponding coupling constant \( \kappa \) is assumed to be purely real, while \( \omega_j = \Omega_j - i \Gamma_j \), \( j = \text{TE}, \text{TM}, P \) are complex frequencies that consist of the resonance frequencies \( \Omega_j \) and the damping coefficients \( \Gamma_j \) (due to radiative and absorptive losses) of the different modes. The antisymmetric matrix

\[
\Delta M = \begin{pmatrix}
0 & 0 & 0 & -\kappa & 0 \\
0 & 0 & 0 & 0 & -\kappa \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}.
\]
$\Delta M$ denotes the nonreciprocal influence of the magnetic field via the Lorentz force proportional to $v \times B$ and is defined as

$$\Delta M = \beta e^{i\theta} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix},$$

(107)

with the real coupling coefficient $\beta$, that is proportional to the static magnetic field. The factor $e^{i\theta}$ stems from the four-oscillator model and represents the phase of the MO oscillator in that system. In the supplement of [41] it is shown by perturbation theory [206, 207] that this phase corresponds to the phase of the gyration $g = |g| \exp(i\theta)$ of the MO material.

The optical response of the system is obtained by assigning an effective susceptibility to the system. This is done by summing up the effective electronic polarization $P_{||} = \chi E_{||}$, which can be identified as $P_{||} = R^T x$. Hence, the effective susceptibility can be written as

$$\chi(\omega) = n R^T M(\omega)^{-1} R,$$

(108)

with $M(\omega) = M_0 + \Delta M - i \omega$. Due to the cross product in the Lorentz force, $\Delta M$ and thus $M(\omega)$ become antisymmetric for non-zero magnetic fields, reflecting the nonreciprocity of the system [4, 48].

The eigenfrequencies of the coupled oscillator system are obtained by setting the external electric field in equation (104) to zero. This results in the following eigenvalue problem

$$(M_0 + \Delta M)x_n = \omega_n x_n,$$

(109)

where $\omega_n$ denotes the eigenvalues, and $x_n$ the eigenvectors for $n = 1, 2, 3$. In the presented model, the Lorentz force is assumed to be weak compared to the restoring forces. Hence, $\Delta M$ is regarded as a small perturbation of $M_0$, resulting in $\omega_n$ being close to the eigenfrequencies of $M_0$. The eigenfrequencies of $M_0$ are given by

$$\omega_{1/2} = \frac{\omega_{TM} + \omega_p}{2} \pm \sqrt{\kappa^2 + \left(\frac{\omega_{TM} - \omega_p}{2}\right)^2},$$

(110)

$$\omega_3 = \omega_{TE}.$$  

(111)

The first two eigenfrequencies correspond to the two branches of a WPP hybrid mode arising from the coupling between the plasmonic mode and the TM waveguide mode [37, 38]. The third eigenfrequency is simply the frequency of the TE polarized waveguide mode. In section 4, it was demonstrated numerically that the largest magneto-optical response occurs for grating periods at which the TM waveguide mode and one of the TM polarized WPP branches possess similar resonance frequencies, i.e., when $\omega_{1/2} = \omega_{TE}$. This behavior can now be deduced analytically from the presented model by examining the inverse of $M$ for a small perturbation $\Delta M$:

$$M^{-1} \approx (M_0^{-1} - i \omega) - \frac{i \beta e^{i\theta}}{(\omega_1 - \omega)(\omega_2 - \omega)(\omega_{TE} - \omega)} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix},$$

(112)

This expression reveals that the magnetic terms proportional to $\beta$ become largest for $\omega = \omega_{TE} = \omega_1/2$, which confirms previous numerical findings.

To obtain the effective susceptibility for a particular nanostructure, the free parameters in $M$ and $R$ have to be deduced by a systematic and rigorous fitting procedure. The fitting process consists of three steps, in which $M_0$, $R$, and $\Delta M$ are fitted sequentially. Full numerical simulations based on the scattering matrix method [192, 208] were used as reference. In the following, the three fitting steps are discussed and applied to a sample geometry that consists of a 150 nm thick EuSe film with 70 nm thick and 70 nm wide gold wires on top (see figure 31(a)). The substrate under the film is assumed to be glass with a permittivity $\varepsilon = 2.13$. The applied magnetic field is assumed to be 5 T. The permittivity of the gold grating is modeled using the Drude model function $\varepsilon(\omega) = \varepsilon_{\infty} - \omega_p^2/(\omega^2 + i\gamma \omega)$, which was introduced in section 2.5.1. The model parameters for gold were extracted from [63] and set to $\varepsilon_{\infty} = 7.9$, $\omega_p = 8.77$ eV, and $\gamma = 1.13 \times 10^{12}$ s$^{-1}$. The components of the permittivity tensor

$$\varepsilon_{EuSe} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ -\varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ -\varepsilon_{13} & -\varepsilon_{23} & \varepsilon_{33} \end{pmatrix}$$

(113)

of the magneto-optical slab are assumed to be $\varepsilon_{11} = \varepsilon_{22} = 4.95 + 0.007i$, $\varepsilon_{33} = 3.9643 + 0.0126i$, $\varepsilon_{12} = 0.061 + 0.061i$, and $\varepsilon_{13} = \varepsilon_{23} = 0$. These values correspond to a 150 nm thick EuSe thin film at a wavelength of 660 nm and a magnetic field of 5 T in $z$-direction. These data were obtained by previous magneto-optical measurements [40].

In the first step, the free parameters in $M_0$ are fitted such that the eigenfrequencies of the mechanical system match the eigenfrequencies of the actual nanostructure obtained by rigorous numerical solutions of Maxwell’s equations. While equation (111) allows the direct derivation of $\omega_{TM}$, the complex coefficients $\omega_{TM}$, $\omega_p$, and the real coefficient $\kappa$ in equation (110) cannot be deduced directly from the full numerical simulations. This can be resolved by calculating the real part of $\omega_{TM}$ from an empty lattice approximation [72] and assuming $\Gamma_{TM} \approx \Gamma_{TE}/10$, as justified by the results by Christ and coworkers [37, 38]. Figure 32 shows the comparison between the simulated and fitted eigenmodes of the oscillator model that includes the Lorentz force. The eigenmodes are plotted as blue (TM) and green (TE) dashed lines in units of wavelength. The solid black lines mark the edge of the light cone (i.e. the Rayleigh anomaly). The blue solid lines correspond to the relevant region around the intersection of the TE waveguide mode and WPP, the modal dispersion is reproduced very well by the oscillator model. Outside this region, in the simulated TM dispersion at around 530 nm, there is a discontinuity due to the presence of another spectrally close higher-order mode. As expected [209], the eigenmodes of the actual nanostructure exhibit a cut-off at the edge of the light cone (see also section 2.6). Since the model does not take the periodic geometry
into account, this discontinuity is missing in the dispersion plot of the model.

In the second part of the fitting sequence, the coefficients in $R$ are derived by the condition that the transmittance of the effective medium has to reproduce that of the actual sample. In analogy to the first fitting step, it is assumed that the magnetic field only has weak influence on the absorption behavior. Hence, the absorption can be derived from the effective susceptibility for zero B-field. This is done by setting $\Delta M = 0$ and solving the Helmholtz equation to obtain the evolution of an $x$- and $y$-polarized plane wave over an effective propagation distance (see section 2.2.2). The effective propagation distance was assumed to be the sample thickness of 220 nm.

For the sake of simplicity, the coefficients in $R$ were assumed to be constant for all grating periods. By comparing the simulated transmittance in figure 32 (left panels) and the modeled transmittance (right panels), it can be seen that, except for the discontinuities outside the region of interest, as discussed above, the line shapes agree very well.

In the last part of the fitting procedure, the remaining magneto-optical coefficients in $\Delta M$ are determined. Far away from the MO material resonance, the gyration $g = |g| \exp(i\theta)$ can be assumed to be constant. For bismuth iron garnet [36] and EuSe [40], this assumption is valid for the red and near-IR spectral region, where $\theta \approx -45$ deg. This is also the value used for the oscillator model and the numerical simulations.

Note that the model will work as well for other materials exhibiting different $\theta$ (see supplemental material in [41]). The last remaining fitting parameter is $\beta$, which is proportional to the magnetic field. For realistic magnetic field strengths, its value only influences the magnitude of the MO response but not its spectral line shape. Thus, $\beta$ is scaled such that the MO response of the modeled system reaches the values of the full simulation.

Figure 33 depicts the comparison of the resulting Faraday rotation spectra from the full numerical simulations and the oscillator model. The green and blue dotted lines trace the eigenmodes. As expected by examining equation (112), the MO response is largest around the intersection point of the TM polarized waveguide plasmon polariton (WPP) branch and the TE waveguide mode. Furthermore, the qualitative and quantitative agreement between model and numerical simulation is excellent.

The corresponding dispersion of the ellipticity response is displayed in figure 34. It can be seen, that the ellipticity predicted by the model is also in good agreement with numerical simulation. As in the case of the Faraday rotation dispersion, it can be seen that the ellipticity response is strongest around the intersection region of the TE waveguide mode and the TM polarized waveguide-plasmon-polariton mode, and lowest far-off resonance.

To compare the exact line shapes of the MO spectra, figure 35 displays the corresponding slice cuts of the Faraday rotation and ellipticity spectra. The model reproduces the line shapes very well. This includes the characteristic up-down feature in the Faraday rotation for $x$-polarization (indicated by arrows), which is successively spreading for periods larger than the period of the TE-WPP intersection (green line) as it was also described in section 4. On the other hand, the modeled MO spectra do not reproduce the offset in the numerically simulated spectra. This is a result of the approximations required for the reduction of the four-oscillators-model to the two-oscillators model. There, the oscillator strength of the MO material was assumed to be much smaller than the oscillator strength of the waveguide modes and the plasmon. As a result,
only the resonant contributions to the MO response are taken into account. The slightly broader MO features result from fitting the resonance linewidths to the values from numerical simulations. A better qualitative agreement in Faraday rotation could be obtained when adjusting the losses in the model independently of the calculated resonance line widths. In summary, it is evident that even the simplified oscillator model correctly predicts the overall shape of the MO spectra very well.

5.2. Simplifications and approximations

In this section, the equation of motion (104) of the simplified oscillator model will be derived explicitly. The discussion will start from the equations of motions of the extended oscillator model with five degrees of freedom (figure 31(b)) described by five coupled second order linear differential equations. By applying appropriate simplifications the system of five equations can be reduced to three coupled quadratic equations, which describe the motion of effectively three oscillators. Furthermore, in order to obtain an analytical solution of the three-oscillator problem, a rotating wave approximation will be applied.

5.2.1. Extended model (5 degrees of freedom). For light propagating in $z$ direction ($E_z = 0$), the equations of motion for the four-oscillators model in figure 31(b) are given by

$$X_{MO} - k_{MO,TM} x_{TM} - k_{m} y_{MO} = \frac{q_{MO}}{m} E_x e^{i\omega t} \quad (114a)$$

$$Y_{MO} - k_{MO,TE} y_{TE} + k_{m} x_{MO} = \frac{q_{MO}}{m} E_y e^{i\omega t} \quad (114b)$$

$$X_{TM} - k_{MO,TM} x_{MO} - k_{p,TM} x_{p} = \frac{q_{TM}}{m} E_x e^{i\omega t} \quad (114c)$$

$$Y_{TE} - k_{MO,TE} y_{MO} = \frac{q_{TE}}{m} E_y e^{i\omega t} \quad (114d)$$

$$X_{p} - k_{p,TM} x_{TM} = \frac{q_{p}}{m} E_x e^{i\omega t} \quad (114e)$$

where the lower case variables $x_j$ and $y_j$ with $(j = MO, P, TE, TM)$ denote the displacements of the oscillators associated with the magneto-optical (MO) material, the plasmon mode (P), as well as the TE and TM waveguide modes in $x$ and $y$ direction. The upper case symbols $X_j$ and $Y_j$ denote terms of the form

$$X_j = \ddot{x}_j + \Omega_j^2 x_j + 2\Gamma_j \dot{x}_j \quad (115a)$$

$$Y_j = \ddot{y}_j + \Omega_j^2 y_j + 2\Gamma_j \dot{y}_j \quad (115b)$$
with \((j = \text{MO, P, TE, TM})\). Dots indicate a time derivative. The coefficient \(k_m\) is proportional to the magnetic field. Due to the cross product in the Lorentz force \(\mathbf{\times} (\mathbf{v} \times \mathbf{B})\), it occurs in the first two equations with opposite signs. In order to reduce the number of parameters in the model, the oscillator masses \(m\) are assumed to be equal for all oscillators. The time harmonic ansatz \(\exp(-i\omega t)\) for the displacements leads to

\[
W_{\text{MO}}(\omega) x_{\text{MO}} - k_{\text{MO,TM}} x_{\text{TM}} + i\omega k_m y_{\text{MO}} = \frac{q_{\text{MO}}}{m} E_x \tag{116a}
\]

\[
W_{\text{MO}}(\omega) y_{\text{MO}} - k_{\text{MO,TE}} y_{\text{TE}} - i\omega k_m x_{\text{MO}} = \frac{q_{\text{MO}}}{m} E_y \tag{116b}
\]

\[
W_{\text{TM}}(\omega) x_{\text{TM}} - k_{\text{MO,TM}} x_{\text{MO}} - k_{p,TM} x_{p} = \frac{q_{\text{TM}}}{m} E_x \tag{116c}
\]

\[
W_{\text{TE}}(\omega) y_{\text{TE}} - k_{\text{MO,TE}} y_{\text{MO}} = \frac{q_{\text{TE}}}{m} E_y \tag{116d}
\]

\[
W_{\text{P}}(\omega) x_{p} - k_{p,TM} x_{p} = \frac{q_{\text{P}}}{m} E_x \tag{116e}
\]

where \(W_j(\omega) = -\omega^2 + \Omega_j^2 = 2i\Gamma_j \omega\), \((j = \text{MO, TM, TE, P})\). In matrix form, equation (116e) reads

\[
M(\omega) \mathbf{x} = \frac{1}{m} Q \mathbf{E}_\parallel. \tag{117}
\]

Here, we introduced the vectors \(\mathbf{x} = (x_{\text{MO}}, y_{\text{MO}}, x_{\text{TM}}, y_{\text{TM}}, x_{p})^T\) as well as \(E_\parallel = (E_x, E_y)^T\) and the matrices

\[
M(\omega) = \begin{bmatrix}
W_{\text{MO}}(\omega) + i\omega k_m & -k_{\text{MO,TM}} & 0 & 0 \\
-i\omega k_m & W_{\text{MO}}(\omega) & 0 & -k_{\text{MO,TE}} \\
-k_{\text{MO,TM}} & 0 & W_{\text{TM}}(\omega) & 0 \\
0 & -k_{\text{MO,TE}} & 0 & W_{\text{TE}}(\omega) \\
0 & 0 & -k_{p,TM} & W_{\text{P}}(\omega)
\end{bmatrix} \tag{118}
\]

and

\[
Q = \begin{pmatrix}
q_{\text{MO}} & 0 \\
0 & q_{\text{MO}} \\
q_{\text{TM}} & 0 \\
0 & q_{\text{TE}} \\
q_{\text{P}} & 0
\end{pmatrix}. \tag{119}
\]

The optical response of the system is obtained by assigning an effective susceptibility to the system. This is done by summing up the effective electronic polarization \(P_\parallel = \chi E_\parallel\), which is given by \(P_\parallel = nQ^T x\), where \(n\) denotes the effective oscillator density. Hence, the effective susceptibility can be written as

\[
\chi(\omega) = \frac{n}{m} Q^T M(\omega)^{-1} Q. \tag{120}
\]

### 5.2.2. Identifying the gyration of the waveguide

The following auxiliary calculation will establish a relation between the gyration of the MO film and the model parameters. The special case of a bare magneto-optical slab is described by equations (116a) and (116b) for \(k_{\text{MO,TM}} = k_{\text{MO,TE}} = 0\). In analogy to equation (117), the corresponding matrix form is

\[
\begin{bmatrix}
W_{\text{MO}}(\omega) + i\omega k_m & \chi_{\text{MO,TE}} \\
-i\omega k_m & W_{\text{MO}}(\omega)
\end{bmatrix} = \frac{q_{\text{MO}}}{m} \begin{pmatrix}
E_x \\
E_y
\end{pmatrix}. \tag{121}
\]

For small \(k_m\), i.e., for small magnetic fields, the resulting susceptibility is then given by

\[
\chi(\omega) = \frac{n q_{\text{MO}}}{m W_{\text{MO}}(\omega)} \begin{pmatrix}
1 + i\omega \frac{k_m}{W_{\text{MO}}(\omega)} \\
-\frac{k_m}{W_{\text{MO}}(\omega)} \frac{1}{1}
\end{pmatrix}. \tag{122}
\]

As discussed in section 2.2.2, for light propagating in z direction, the susceptibility tensor \(\chi = \epsilon - i\eta\) of a MO material can be written as

\[
\chi = \begin{pmatrix}
\chi_{xx} & +i\gamma \\
-i\gamma & \chi_{yy}
\end{pmatrix}, \tag{123}
\]

where \(g\) is the complex magnetic field induced gyration of the material. By comparing the equations (122) and (123) \(g\) can be identified to be

\[
g = |g| e^{i\theta} = -\frac{n q_{\text{MO}}}{m W_{\text{MO}}(\omega)^2}. \tag{124}
\]

### 5.2.3. Simplified model (3 degrees of freedom)

Solving equations (116a) and (116b) for \(x_{\text{MO}}\) and \(y_{\text{MO}}\) yields in the limit of a small magnetic influence (i.e. \(k_m \ll W_{\text{MO}}\))

\[
x_{\text{MO}} = + \frac{1}{W_{\text{MO}}(\omega)} (m^{-1} q_{\text{MO}} E_x - k_{\text{MO,TM}} x_{\text{TM}}) - i\omega k_m \frac{\omega}{W_{\text{MO}}(\omega)^2} (m^{-1} q_{\text{MO}} E_x - k_{\text{MO,TE}} y_{\text{TE}}) \tag{125}
\]

\[
y_{\text{MO}} = + \frac{1}{W_{\text{MO}}(\omega)} (m^{-1} q_{\text{MO}} E_y - k_{\text{MO,TE}} y_{\text{TE}}) + i\omega k_m \frac{\omega}{W_{\text{MO}}(\omega)^2} (m^{-1} q_{\text{MO}} E_x - k_{\text{MO,TM}} x_{\text{TM}}). \tag{126}
\]

The equations (125) and (126) can now be inserted into the equations (116c) and (116d) to eliminate \(x_{\text{MO}}\) and \(y_{\text{MO}}\). It is reasonable to assume that the MO oscillator couples only weakly to the far field and that the system is mainly driven via the waveguide and the plasmon oscillators. This means that the driving forces of the MO oscillator are assumed to be much weaker than the internal coupling forces \((m^{-1} q_{\text{MO}} E_x) \ll |k_{\text{MO,TM}} x_{\text{TM}}|\) and \(|m^{-1} q_{\text{MO}} E_y| \ll |k_{\text{MO,TE}} y_{\text{TE}}|\). In this limit and by using the equation (124), the equations (116c) and (116d) can be rewritten as

\[
(-\omega^2 + \Omega_{\text{TM}}^2 - 2i\Gamma_{\text{TM}} \omega - \frac{k_{\text{MO,TM}}^2 W_{\text{MO}}(\omega)}{m}) x_{\text{TM}}
\]

\[
- ig \frac{mk_{\text{MO,TM}} k_{\text{MO,TE}}}{nq_{\text{MO}}} y_{\text{TE}} - k_{p,TM} x_{p} = m^{-1} q_{\text{TM}} E_x \tag{127}
\]

\[
(-\omega^2 + \Omega_{\text{TE}}^2 - 2i\Gamma_{\text{TE}} \omega - \frac{k_{\text{MO,TE}}^2 W_{\text{MO}}(\omega)}{m}) y_{\text{TE}}
\]

\[
+ ig \frac{mk_{\text{MO,TM}} k_{\text{MO,TE}}}{nq_{\text{MO}}} x_{\text{TM}} = m^{-1} q_{\text{TE}} E_y. \tag{128}
\]
Together with (116e), these two equations fully describe the optical system. Hence, the applied approximations led to a reduction to only three equations. Usually, the material resonance of the MO slab is far away from the excitation frequency and the waveguide frequencies (i.e., $\Omega_{MO} \gg \Omega_{TM}, \Omega_{TE}, \omega$), which means that the terms proportional to $1/W_{MO}(\omega)$ can be neglected. This results in the following set of equations:

\[
\begin{align*}
-\omega^2 + \Omega_{TM}^2 - 2i\Gamma_{TM}\omega)x_{TM} & = m^{-1}q_{TM}E_i, \\
-\omega^2 + \Omega_{TE}^2 - 2i\Gamma_{TE}\omega)y_{TE} & = m^{-1}q_{TE}E_i, \\
-\omega^2 + \Omega_{P}^2 - 2i\Gamma_{P}\omega)x_{P} & = m^{-1}q_{P}E_i, \\
\end{align*}
\]

(129a)

(129b)

(129c)

where $b = |g|mk_{MO,TM,TE}/n^2_{MO}$ and $\theta = \arg(g)$. For an absent driving electrical field, i.e., for $E_i = 0$, the equations above represent a quadratic eigenvalue problem for the eigenfrequencies $\omega$. Obtaining these eigenfrequencies would therefore require to find the roots of a polynomial with degree six, which cannot be accomplished analytically. However, by applying a rotating wave approximation, the quadratic eigenvalue problem can be reduced to a linear eigenvalue problem. This approximation is valid, when the relevant frequencies are in the same range, that is:

\[\begin{align*}
-\omega^2 + \Omega_j^2 \approx (\Omega_j - \omega)2\bar{\Omega}_j, \\
\end{align*}\]

Here, $\bar{\Omega}$ is a constant average frequency close to $\omega$, $\Omega_{TM}$ and $\Omega_{TE}$. In this approximation, the equations of motion become:

\[
\begin{align*}
(\omega_{TM} - \omega)x_{TM} - i\beta e^{i\theta}y_{TE} - \kappa x_{P} & = \eta\rho_{TM}E_i, \\
(\omega_{TE} - \omega)y_{TE} + i\beta e^{i\theta}x_{TM} & = \eta\rho_{TE}E_i, \\
(\omega_{P} - \omega)x_{P} - \kappa x_{TM} & = \eta\rho_{P}E_i, \\
\end{align*}
\]

(131a)

(131b)

(131c)

The following definitions were used: $\beta = b/2\bar{\Omega}$, $\kappa = k_{P,TM,TE}/2\Omega$, $\eta = 1/2m\bar{\Omega}\Omega$, $\rho_j = nq_j$ and $\omega_j = \Omega_j - i\Gamma_j$, with $j = TM, TE, P$, where $n$ is the oscillator density in the effective medium. When writing the equations above in the matrix form

\[
(M_0 + \Delta M - I\omega)\mathbf{x} = \eta R E_i ,
\]

(132)

with $\mathbf{x} = (x_{TM}, x_{P}, y_{TE})^T$ and

\[
M_0 = \begin{pmatrix}
\omega_{TM} & -\kappa & 0 \\
-\kappa & \omega_{P} & 0 \\
0 & 0 & \omega_{TE}
\end{pmatrix},
\]

(133)

\[
\Delta M = \beta e^{i\theta} \begin{pmatrix}
0 & 0 & -i \\
0 & 0 & 0 \\
i & 0 & 0
\end{pmatrix},
\]

(134)

\[
R = \begin{pmatrix}
\rho_{TM} & 0 & 0 \\
\rho_{P} & 0 & 0 \\
0 & \rho_{TE}
\end{pmatrix},
\]

(135)

it is evident that we arrived at the equation of motion as utilized in section 5.1.

5.3. Conclusion

In summary, the dispersion of the MO response of hybrid magnetoplasmonic waveguides can now be understood in the picture of a simple oscillator model including the nonreciprocal Lorentz force. In the case of a weak influence of the Lorentz force, analytical expressions for the optical response were obtained, which confirm previous numerical findings. Importantly, the spectral line shape of the MO response is fully determined by the optical properties of the system for zero magnetic field. Only the overall magnitude of the MO response is determined by the applied magnetic field.

The theory in this section provides the understanding required to further develop hybrid magnetoplasmonic systems in highly integrated optics, demanding actively controlled optical modulation [195], magnetic field sensing [196, 197], refractive index sensing [143], and optical isolation [12–14, 16–18]. It should also be mentioned that the presented findings can not only help to understand and optimize existing sample geometries, but can also be transferred to other geometries, such as 2D plasmonic gratings. Furthermore, by removing all plasmonic oscillators, the case of a purely dielectric grating-waveguide combination can be realized.

In section 6 the presented model will be applied to a system consisting of an EuS thin film and an embedded plasmonic grating. By means of the model, it will be shown, that in the case of weak coupling between the plasmonic and waveguide resonance, the classical optical analog of electromagnetically induced transparency and absorption can be realized. Moreover, the model will be of fundamental importance to understand why the corresponding magneto-optic response in such a system is so exceptionally large.

In prospect of non-linear magnetoplasmonics, the presented model could also be of fundamental relevance. Although the described model is fully linear, a non-linear extension would be straight-forward by adding higher order coupling terms [210].

6. Giant Faraday effect via induced absorption

In this section a new hybrid magnetoplasmonic thin film structure will be demonstrated, that resembles the classical optical analog of electromagnetically induced absorption. In transmission geometry the gold nanostructure embedded in an EuS film induces giant Faraday rotation of up to $14^\circ$ for a thickness of less than 200 nm and a magnetic field of 5 T at $T = 20$ K. By varying the magnetic field from $-5$ T to $+5$ T, a rotation tuning range of over $25^\circ$ is realized. As this is only a factor of three away from the Faraday isolation requirement, the demonstrated concept could lead to highly integrated, nonreciprocal photonic devices for optical isolation, light modulation, and optical magnetic field sensing.

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6.1. Realization with EuS-Au hybrid systems

In the last two sections it was demonstrated how the magnetooptic (MO) response of a dielectric thin film is enhanced by the attachment of a plasmonic grating. This technique also allows the amplified MO response to be spectrally tailored by tuning the grating parameters. However, the MO performance of such a system is limited by the low $Q$ factor of the plasmon resonances in the grating. In the following, it will be shown that this limitation can be lifted elegantly by a novel approach, which is based on a classical optical analog of electromagnetically induced absorption (EIA) [181, 182]. Here, a strongly damped plasmon oscillation is weakly coupled to a narrow linewidth waveguide resonance with a phase delay, leading to constructive interference. By tuning this coupling carefully, a high-$Q$ absorptive hybrid mode is realized, which can be used to resonantly amplify the Faraday rotation response. Furthermore, the EIA mechanism allows to utilize the high oscillator strength of the plasmonic resonance, leading to an efficient coupling of the incident light into the structure without reducing the effective $Q$ factor due to the broad plasmonic resonance as was the case in previous approaches [34, 36, 40]. Although being only less than 200 nm thick, the presented novel structure design exhibits an order of magnitude better rotation capability than previous approaches that only resulted in fractions of degree rotation [24–26, 36, 63].

Furthermore, for low magnetic fields, the presented technique still yields rotation angles that were only achievable with 20 times stronger field strength in previous approaches [40]. The performance of the thin film structure is only a factor of 3 away from the 45° that are required for building a Faraday isolator which usually requires MO media with a thickness on the order of centimeters. The first demonstration of plasmonic resemblances of EIA [211, 212] and its related effect, namely electromagnetically induced transparency (EIT) [213–215], triggered significant attention. The special dispersion properties of such systems are known to facilitate plasmonic sensing with narrow linewidths [216–218] as well as slow light [219–221] and delay lines [222, 223]. Furthermore, there have been extensive studies on enhanced nonlinear response [224, 225] in such systems. Here, it will be demonstrated that the EIA-like optical dispersion in the presented system also facilitates a giant MO response. While in EIT the light within the narrow spectral band passes through the sample, in the case of EIA real material polarization is excited within a narrow spectral region.

Another important aspect of the presented approach is the introduction of EuS as a new dielectric material for hybrid magnetoplasmonics. At low temperatures, EuS possesses an exceptionally large Verdet constant in combination with a high saturation magnetic flux density, resulting in a potentially very strong MO response [58, 194]. Furthermore, it is transparent in the visible, qualifying it for the utilization in Faraday geometry. While EuS possesses as similar refractive index and absorption coefficient as EuSe [58, 194], EuS is significantly cheaper and already exhibits a stronger MO response at low magnetic field strengths. Further information about the temperature dependence of the EuS material parameters can be found in the supplemental material of [42] as well as in the [53, 58, 194, 226]. We note in passing that in the utilized temperature regime the changes in the optical constants of both gold [227] and the fused silica substrate [228] can be neglected. Despite the need for low temperatures, EuS is well suited for studying plasmonic MO model systems, as it combines the above-mentioned benefits with simple fabrication by physical vapor deposition. This allows for the realization of complex hybrid magnetoplasmonic structure geometries, which are challenging when fabricated with commonly used magneto-optic materials, such as bismuth iron garnet [60] or yttrium iron garnet [229]. The reason is that the deposition of garnet films is a sophisticated process that usually involves pulsed laser deposition and high-temperature annealing that could damage any underlying plasmonic structures.

The utilized structure geometry is depicted in the figures 36(a) and (b). It consists of an EuS thin film without an embedded Au nanowire grating. The EuS film is magnetized by an external magnetic field in the $z$ direction, which is also the direction of propagation of the incident light. The structure is fabricated in a three-step process. First, the bottom layer of the EuS film with thickness $b$ is evaporated onto the glass substrate by physical vapor deposition. After that, the gold wire grating with thickness $t$, width $w$, and period $p$ is structured via electron beam lithography. In the last step, an Au film with thickness $(h-b)$ is evaporated. This results in an EuS-Au thin film structure with a slightly corrugated upper surface. Figure 36(c) shows a colored scanning electron micrograph of the cross section of such a structure with geometry parameters $p = 490$ nm, $t = 33$ nm, $w = 85$ nm, $h = 139$ nm, and $b = 33$ nm. For the case of $b = 0$, the non-MO dispersion properties of such a metal-dielectric hybrid structure have previously been analyzed in detail by Zentgraf et al [230]. Because of the presence of the glass substrate
Figure 37. (a),(b) Simulated transmittance and absorbance 
\((1 - T - R)\) with \(T\) and \(R\) denoting transmittance and reflectance, respectively) for \(x\)-polarized incident light and different burial parameters \(b\). For increasing \(b\) there is a gradual transition from the regime of induced transparency to induced absorption. The white dotted line indicates the Rayleigh anomaly. (c),(d) Slice cuts for a clearer view on the line shapes of the spectra. The oval indicates the EIA regime, characterized by very sharp induced absorption peaks. Reproduced from [42]. CC BY 4.0.

with a higher refractive index than the air above the sample, the \(x\) component of the electric field of the TM waveguide mode is concentrated in the upper part of the EuS waveguide. Hence, the coupling between plasmon and TM waveguide mode is weak in comparison to the case where the metal wires are attached on top. In the work by Zentgraf et al it was also demonstrated that this weak coupling can lead to the classical analog of the quantum mechanical effect of electromagnetically induced transparency, resulting in a narrow transmission peak on top of a broad transmission dip for \(x\)-polarized incident light.

6.1.1. Exploiting the regime of induced absorption. It will now be demonstrated that by increasing the burial parameter \(b\) in the hybrid system, the phase between the plasmonic mode and the TM waveguide mode can be tuned such that the system undergoes a transition from EIT-like to EIA-like behavior [211, 212]. Figure 37 displays the simulated transmittance and absorbance spectra for \(x\)-polarized incident light and different burial parameters \(b\). The simulations have been carried out using the Fourier modal method for anisotropic materials [231]. The implementation is based on a scattering matrix algorithm [72], which has been improved by adaptive spatial resolution [208] to enable an efficient derivation of optical properties of metallodielectric systems. The refractive index of the substrate is assumed to be 1.456, and for the gold wires the permittivity data by Johnson and Christy [193] are used. In the investigated wavelength range, the refractive index of EuS is approximately 2. The utilized material model parameters for EuS can be found in the supplemental material of [42]. The geometry parameters were set to \(p = 490\) nm, \(t = 33\) nm, \(w = 85\) nm, and \(h = 139\) nm. In all simulations the bumps in the EuS film above the Au wires are approximated by a rectangular shape with thickness \(t\) and a width of \(2w\). As expected [230], for small values of \(b\), the transmittance spectrum exhibits the characteristic EIT line shape with a broad, mainly plasmon-induced dip and a narrow, mainly TM waveguide-induced peak. For increasing values of \(b\), this effect is reversed and the sharp waveguide feature flips around, resulting in a sharp absorbance peak that is characteristic for an EIA-like system. At 715 nm, the spectra are slightly distorted due to the Rayleigh anomaly [38]. The transition of the spectra with increasing \(b\) can be understood in the picture of coupled Lorentz oscillators introduced by Taubert et al [212]: here, a strongly damped Lorentz oscillator (in this case the plasmon) and a less damped Lorentz oscillator (in this case the TM waveguide mode) are coupled via a complex coupling constant that includes an additional phase delay. It was shown that by changing the phase of the coupling constant, the optical response of the system turns from EIT-like to an EIA-like behavior. In the present case, this behavior is particularly pronounced for \(b\) between 25 and 36 nm (see figure 37(d), oval area). Furthermore, by changing the distance between the wire grating and the substrate, the coupling phase can be changed. As in other cases of plasmonic EIT and EIA, the reflection behavior of the system also changes with the coupling phase, which leads to the situation that the EIT coupling regime can be observed most pronounced in the transmittance spectrum, whereas the EIA case can be identified best in the absorbance spectrum [212]. In section 6.2, there will be an extended discussion on interpreting the presented magnetoplasmonic system in the picture of coupled oscillators.

For a burial parameter of \(b = 33\) nm, the EIA coupling regime manifests itself also in the transmittance spectrum in the form of a small dip (see figure 37(c)). For this case the relation between the EIA-like behavior and the MO response was investigated experimentally. The figures 38(a) and (b) display the measured transmittance and Faraday rotation spectra for \(x\)-polarized incident light and different grating periods \(p\). The measurements are performed at 20 K and a magnetic field of 5 T in the \(z\) direction. The Faraday rotation was measured with a rotating analyzer setup where the incident polarization state was prepared using a polarizer inside the cryostat. For a period of 490 nm, the sharp EIA feature is best centered with respect to the broad, mainly plasmonic, transmittance feature. For the same grating period, at the spectral position of the EIA resonance, the Faraday rotation exhibits a sharp maximum of about \(8^\circ\), which is a substantial rotation enhancement compared to a bare EuS film with thickness \(h = 139\) nm (gray curve). This behavior can be understood in the following way: For wavelengths close to the narrow EIA resonance, the system acts as a resonator.
with a relatively high $Q$ factor, implying multiple vertical round-trips of the light through the MO material before it is coupled out in z direction. Due to Lorentz non-reciprocity, the polarization rotation adds up for each propagation cycle, resulting in an enhanced rotation compared to a single pass through the EuS film. Furthermore, the EIA mechanism allows utilizing the high oscillator strength of the plasmonic resonance, leading to an efficient coupling of the incident light into the structure without reducing the effective $Q$ factor due to the broad plasmonic resonance as was the case in previous approaches [34, 36, 40]. As such, the plasmonic EIA mechanism provides an elegant way to greatly increase the interaction between the incident light and the MO material. In section 6.2 the connection between the EIA dispersion and Faraday rotation enhancement will be discussed in further detail by means of a coupled oscillator model. The measurement is also in good agreement with the full numerical simulation displayed in the figures 37(c) and (d). In the measured wavelength region, the Faraday rotation of EuS shows a notable wavelength dependence due to a material resonance (see gray line). Hence, in order to obtain realistic simulation results, the material dispersion of EuS is modeled with a MO Lorentz oscillator (see section 2.3.1). The free model parameters are fitted such that the measured MO response of a bare EuS film matches the simulated MO response in the wavelength range of interest between 650 and 850 nm. The discrepancy between the measured and simulated Faraday rotation below 650 nm is due to the limited model accuracy in this range. In the transmittance spectra, the grating-induced Rayleigh anomaly is barely visible. For example, for $p = 490$ nm it occurs at 715 nm and is most visible in the simulated Faraday rotation, where it manifests itself as a small kink. However, the Rayleigh anomaly has no major influence on the Faraday rotation enhancement as it is well separated from the enhancement by the EIA resonance.

Also for $y$-polarized incident light, large Faraday rotation angles can be realized. This is illustrated by the figures 39(a) and (b), which depict the corresponding transmittance and Faraday rotation spectra for a sample geometry with $t = 33$ nm, $w = 75$ nm, $h = 139$ nm, $b = 33$ nm, and different grating periods $p$. Since a direct excitation of the plasmon resonance and TM waveguide resonance is only possible for $x$-polarized light, they have no discernable influence on the transmittance line shape. However, as the $x$- and $y$-polarized eigenmodes are coupled via the magnetic field, the Faraday rotation spectrum for $y$-polarized incidence is strongly influenced by the $x$-polarized eigenmodes (and vice versa) [41], which results in strongly enhanced Faraday rotation also for $y$-polarized incidence. Furthermore, this behavior will be discussed in more detail in section 6.2. For a grating period of 505 nm, the Faraday rotation reaches values of up to 14° at a transmittance value of 17%. Given the thin structure profile of below 200 nm, this is a giant Faraday rotation value and only a factor of 3 away from a Faraday isolator. Again, the measurement results are in good agreement with the numerical simulations plotted in the figures 39(c) and (d), except that the Rayleigh anomalies (kinks in the left shoulders of the transmittance spectra) are washed out in the measurement. This happens certainly due to sample roughness and limited fabrication accuracy. As in the case of $x$-polarized incidence, the Rayleigh anomalies are well separated from the Faraday rotation enhancement feature and have no significant influence on the Faraday rotation spectra.

6.12. Influence of magnetic field strength. Furthermore, since the rotation angle scales with the applied magnetic field, the presented structure offers an impressive polarization tuning range of over 25°. This is indicated in figure 40(a), where the Faraday rotation spectra are plotted for different magnetic
field strengths ranging from $-5$ to $+5$ T. The spectra for inverted magnetic fields exhibit almost perfectly mirror symmetric shapes. Any deviations from absolute mirror symmetry can be explained by a slight misalignment between the gold wires and the incident polarization. Figure 40(b) shows a close-up of the Faraday rotation enhancement feature for lower magnetic field strengths that can be realized easily using standard permanent magnets [232]. Even for magnetic fields as low as 250 mT, the Faraday rotation still reaches values of up to 4.9°. This is a similar rotation performance as was previously realized for a 220 nm thick hybrid structure based on EuSe, however, only with 20 times larger magnetic fields [40]. Figure 40(c) displays the Faraday rotation at 737 nm as a function of applied magnetic field and illustrates the saturation behavior. Up to 500 mT the rotation angle increases linearly with the applied magnetic field, whereas for larger fields saturation sets in and the response curve flattens out.

### 6.1.3. Simulation of the ellipticity spectra

As the utilized measurement setup does not allow for measuring the ellipticity spectra, in the following, the numerically simulated ellipticity spectra are provided. Figure 41 displays the ellipticity spectra of the sample geometries with 85 and 75 nm wire width. In the first case, a grating period of 490 nm results in the largest Faraday rotation peak. At its center wavelength, the corresponding ellipticity is close to zero. This behavior is consistent with the following rule of thumb, originating from the Kramers–Kronig relations [44]: a symmetric Faraday rotation peak usually corresponds to asymmetric (s-shaped) ellipticity spectrum with a zero-crossing at the center wavelength of the rotation peak. On the other hand, a symmetric ellipticity peak corresponds to s-shaped Faraday rotation spectrum with the zero crossing at the center wavelength of the ellipticity peak. For example, the latter behavior is exhibited for a wire width of 75 nm, and a period of 505 nm, as illustrated in figure 41(b).

### 6.2. Harmonic oscillator modeling

To obtain further insight into the connection between EIA-like modal coupling and the enhancement of the magneto-optic response, it is useful to view the presented magnetoplasmonic system in the picture of coupled oscillators, as depicted in figure 42(a). This oscillator scheme was originally introduced in [41] and is now modified in analogy to the non-magneto-optic model by Taubert et al to account for EIA [212]. In this very simple model, the magnetoplasmonic system is described using two charged oscillators that are driven by the external light field: the oscillator on the right-hand side represents the plasmonic excitation, and its movement is restricted to the x-direction, as the plasmonic resonance can only be excited for x-polarized light. The second oscillator is associated with the waveguide excitations inside the MO slab. This oscillator can be displaced both in the x and y direction, which corresponds to the excitation of TM and TE polarized waveguide modes respectively. As discussed by Taubert et al, it is crucial for realizing the coupling regime of EIA that the coupling between the two contributing oscillators includes a retardation phase [211]. Hence, the coupling between plasmon and TM waveguide is modeled with a complex coupling constant $\tilde{\kappa} = \kappa \exp(i\phi)$, with $\phi$ as the phase. The influence of the applied static magnetic field is taken into account via the Lorentz force acting on the 2D waveguide oscillator. The resulting electric susceptibility associated with this oscillator system is given by [41]

$$\chi(\omega) = \begin{pmatrix} \chi_x & +\chi_{xy} \\ -\chi_{xy} & \chi_y \end{pmatrix} = \eta R^2 M^{-1} R,$$  \hspace{1cm} (136)
where the matrices
\[
M = \begin{pmatrix}
\Omega_{TM} & -\kappa e^{i\phi} & -i\beta \\
-\kappa e^{i\phi} & \Omega_p & 0 \\
i\beta & 0 & \Omega_{TE}
\end{pmatrix}
\]

and
\[
R = \begin{pmatrix}
\rho_{TM} & 0 & 0 \\
0 & \rho_p & 0 \\
0 & 0 & \rho_{TE}
\end{pmatrix}
\]

are introduced. Here, the terms \( \Omega_i = \omega_i - \omega - i\gamma_i/2 \), \( i = \text{TM, P, TE} \) contain the eigenfrequencies and damping coefficients of the individual oscillators. The magneto-optic coefficient \( \beta \) is proportional to the gyration of the waveguide material and, thus, to the applied magnetic field. The occurrence with two different signs originates from the cross product in the Lorentz force. The nonzero components \( \rho_i \) \( i = \text{TM, P, TE} \) in the matrix \( R \) are proportional to the charge of the oscillators and relate to the individual oscillator strengths. \( \eta \) is a proportionality constant.

In the following, this model is used to illustrate the relation between the EIA and EIT coupling regimes as well as their influence on the magneto-optic response. For this, to keep the degrees of freedom to a minimum, the following simple (and dimensionless) parameter constellation is utilized: the individual resonances in the system are spectrally close, while the waveguide modes have significantly smaller linewidths than the plasmon. Hence, the eigenfrequencies of the three oscillators are assumed to be equal, i.e. \( \omega_{TM} = \omega_{TE} = \omega_p = \omega_0 \) and for the damping coefficients it is assumed that \( \gamma_p = 1 \) and \( \gamma_{TM} = \gamma_{TE} = 0.1 \). Furthermore, the proportionality constant \( \eta \) is set to 1 and it is assumed that the plasmon oscillator couples much stronger to the external light field than to the TM waveguide mode. Hence, the oscillator strengths are weighted by setting \( \rho_{TM} = 0 \) and \( \rho_p = 1 \). Furthermore, the oscillator strength of the TE waveguide oscillator was set to \( \rho_{TE} = 0.32 \), which represents the case in which \( \chi_x \) and \( \chi_y \) (not shown here) are approximately equal.

The plots in figure 42(b) show the resulting frequency dependence of the imaginary part of \( \chi_x \), which is proportional to the absorbance of \( x \)-polarized incident light. The different line colors correspond to the different coupling phases. Furthermore, from the left to the right column, the coupling amplitude \( \kappa \) is decreased. All spectra of \( \chi_x \) show a broad, mainly plasmon-induced background with a sharp modulation on top. For \( \phi = 0 \), the modulation is a narrow dip at \( \omega_0 \) and corresponds to the case of EIT. Increasing the coupling phase to \( \phi = \pi/2 \) turns this modulation into a narrow EIA resonance, while the case of \( \phi = \pi/4 \) represents an intermediate regime. Furthermore, an increase of \( \kappa \) leads to a more pronounced modulation. However, at some point when \( \kappa \) is increased further (not shown here), the coupling is not weak anymore and a significant mode splitting occurs for \( \phi = 0 \), and for \( \phi = \pi/2 \) solutions with negative absorption can emerge. As such solutions correspond to the the presence of gain, they are not relevant for the passive plasmonic system discussed here [212]. In summary, the classical analogs of EIT and EIA are indeed very closely related: both scenarios occur in the regime of weak coupling where there is no significant spectral repulsion between the narrow linewidth mode (in this case, the TM waveguide mode) and the broad linewidth mode (in this case, the plasmon). The only difference between the two scenarios lies in the coupling phase \( \phi \). It should be pointed out that the absorption behavior is approximately independent of the magnetic field, as \( |\beta| \) can be assumed to be small (here, it is set to 0.001), in which case it predominantly influences the off-diagonal elements of \( \chi \) but not the diagonal elements [41].

The corresponding magneto-optic response is encoded in the off-diagonal elements of \( \chi \) that are proportional to \( \beta \) and, thus, to the magnetic field. In first approximation, the Faraday rotation of the system is proportional to the imaginary part of \( \chi_{xy} \) (see section 2.2.3), which is plotted in figure 42(c). It should be noted that the real part of \( \chi_{xy} \) would correspond to the ellipticity (not plotted here). The graphs nicely illustrate the fundamental working principle of the system: with increasing \( \kappa \), not only the modulation of \( \chi_x \) increases (i.e. the EIA or EIT resonances become more pronounced), but also the modulation of \( \chi_{xy} \) becomes stronger. In other words, due to the magnetic field, the EIA or EIT resonances not only occur in absorption, but also in Faraday rotation. The more pronounced the EIA or EIT resonances are, the larger the Faraday rotation enhancement becomes. Furthermore, for a constant \( \kappa \) this modulation of

![Diagram](image-url)
$\chi_{xy}$ is strongest for $\phi = \pi/2$, i.e. for the EIA case. At this point it should be noted that in practice the difference in the MO response obtained by EIA-like and EIT-like coupling can be less pronounced than in the modeled spectra. The reason is that tuning the structure geometry from the EIT to the EIA case usually also slightly influences the other coupling parameters such as $\kappa$, which also have a strong influence on $\chi_{xy}$. However, the model correctly reflects the trend that the EIA case produces stronger MO response than the EIT case. Another revealing aspect of this model is that although only the oscillators moving in the $x$ direction contribute to the EIA resonance, this resonance also leads to increased polarization rotation for $y$-polarized light: this follows from the fact that $\chi$ is antisymmetric. Hence, the resonance in $\chi_{xy}$ translates to a conversion from $x$ to $y$ polarization as well as to a conversion from $y$ to $x$ polarization. However, in general, the Faraday rotation spectra for $x$- and $y$-polarized light are not completely equal since for finite propagation distances also the diagonal components $\chi_{xx}$ and $\chi_{yy}$ possess an influence [41, 44]. As shown by the measurements in figure 39, the Faraday rotation can be even larger for $y$-polarized light than for $x$-polarized light. It should also be pointed out that the line shape of $\chi_{xy}$ depends on the complex phase of the magneto optical coupling constant $\beta$, which is related to the gyration of the MO material [41]. For the present example, this phase is set to $\sim -45^\circ$. Changing this phase does not affect the average magnitude of the magneto-optic response, but only determines whether the Im($\chi_{xy}$) spectra show a down-up line shape (negative phase), an up-down line shape (positive phase), or a peak (zero phase) when plotted over angular frequency. Due to the $\omega$ based mathematical structure of the model, the response curves were plotted over angular frequency. This should be kept in mind when the modeled spectra are compared to the measured spectra, which are plotted over wavelength.

As a final remark, it should be mentioned that the presented oscillator model is the simplest possible approach to illustrate the relation between EIA-like coupling and the enhanced MO response in the system. Of course, this results in neglecting secondary influences observed in the actual measurements, such as material dispersion, background Faraday rotation due to the film, or diffraction effects such as Rayleigh anomalies [41]. Nevertheless, the benefit of this model is that it provides a very intuitive description of the system.

6.3. Conclusion

The design flexibility enabled by the use of EuS as magneto-optic material was exploited to realize a hybrid magnetoplasmonic thin film structure that represents the classical optical analog of EIA. Unlike in previous approaches where the low quality factor of localized surface plasmon resonances limited the Faraday rotation enhancement, the coupling regime of EIA allows us to leverage both the high quality factor of the waveguide resonances and the large oscillator strength of the plasmons. Both these aspects result in dramatically increased light-matter interaction and MO response. As a result, the EIA-like system induces giant Faraday rotation of over $14^\circ$ for a thickness of less than 200 nm and a magnetic field of 5 T at $T = 20$ K. By varying the magnetic field from $-5$ T to $+5$ T, an impressing rotation tuning range of over $25^\circ$ is realized. This is a MO performance that exceeds any previous approach considerably.

7. Conclusion and outlook

We have seen that the field of nonreciprocal plasmonics has become big and multifaceted. Furthermore, every method of breaking Lorentz reciprocity usually comes with certain practical limitations, such as fabrication difficulties, optical non-linearities, size constraints and overall technical feasibility. The various approaches in nonreciprocal nanophotonics can be divided into two groups: firstly, there have been efforts to realize enhanced MO effects of nanostructures with the aim to eventually integrate them into miniaturized versions of conventional nonreciprocal devices such as Faraday isolators. Secondly, there have been several attempts to directly realize complete nonreciprocal photonic devices such as optical isolators and circulators.

Our analysis of the second group of approaches showed that, so far, there has not been realized a system with practical degrees of isolation and low losses simultaneously. Especially nonlinear systems seem inherently hampered as nonlinear effects are usually exploited through losses. Furthermore, the intensity dependent optical isolation is usually not practical. Time modulated systems are very promising, as they already reached practical relevance within the microwave regime. However, optical realizations are usually only theoretical or proof-of-concept realizations [8]. Nevertheless, the field still becomes more and more active.

With regards to enhanced Faraday rotation we have seen that dielectric MO films in combination with plasmonic enhancement have been proven to be a very powerful approach. In comparison, ferromagnetic nanoantennas are hampered by large optical losses and significantly lower MO response due to low quality resonances. However, magnetoplasmonic antennas remain a very intriguing platform for sensing applications [145]. Furthermore, as the required fabrication techniques are widely available, magnetoplasmonic antennas are also attractive for studying fundamental MO interaction effects such as chirality in combination with magnetic gyration. For such studies it is beneficial that magnetoplasmonic antennas comprise a rather simple and clean physical system.

In this review we demonstrated that especially large Faraday rotation can be achieved for hybrid magnetoplasmonic crystals employing one of the two working principles: In the first case, a gold nanowire grating is attached on top of a magneto-optic (MO) thin film. The strong coupling between the plasmon resonance in the gold grating and a waveguide mode in the MO film results in the formation of a waveguide-plasmon-polariton (WPP). This hybrid mode can facilitate large Faraday rotation of up to $4^\circ$ for a $220$ nm thick EuSe-Au structure. Furthermore, the WPP dispersion properties can be leveraged to tailor the working wavelength of a Faraday rotator via the
grating parameters. However, even stronger MO response can be obtained for the second type: here, a plasmonic gold grating is embedded into a MO film. Such a system can be regarded as a classical optical analog of electromagnetically induced absorption (EIA). Its unique dispersion properties allow to compensate for the low $Q$ factor of the plasmonic resonance, which limits the MO response in the WPP-based approach. The presented EIA-like EuS-Au structures induce giant Faraday rotation of up to $14^\circ$ for a thickness of less than 200 nm and a magnetic field of 5 T at $T = 20$ K. Furthermore, by varying the magnetic field from $-5$ to $+5$ T, the polarization of the transmitted light can be continuously tuned over an impressively wide range of over $25^\circ$. As the realized magneto-optic response is only a factor of three away from the Faraday isolation requirement, this concept could lead to highly integrated, nonreciprocal photonic devices for optical isolation, light modulation, and optical magnetic field sensing.

Furthermore, as was demonstrated recently, strong MO dispersion of nanostructured systems can also be leveraged for refractive index sensing, allowing very precise detection of biochemical substances [145]. Here, the exceptionally sharp spectral features in the MO response of EIA systems could also turn out to be a powerful tool.

A fruitful strategy to even further increase the MO response of the EuS-Au structures could be to use a thicker MO slab in combination with gratings that possess a similar lattice constant as in the case of the EIA structures discussed in section 6. This way, higher-order waveguide resonances could be excited, which, in combination with EIA-like dispersion, could lead to Faraday rotation values closer to or even above $45^\circ$. Also some sort of physical or chemical treatment to enhance the surface quality of the EuS film is expected to further increase the resonator performance resulting in larger Faraday rotation.

In future designs, the necessity for low temperatures could be significantly relaxed by doping europium compounds with gadolinium. For example, it has been shown that doping EuO with gadolinium can raise the Curie temperature (where the Faraday rotation starts to drop) from 69 to 135 K [57, 205], which is well above liquid nitrogen temperature. Of course, the concept of WPP- and EIA-boosted MO response is not restricted to Eu compounds and can in principle be applied to many materials. The only requirements are a sufficient transparency of the MO material and a suitable procedure for the nanostructure fabrication. For example, although BIG only provides moderate MO response, it allows room-temperature operation and exhibits high transparency in the near-infrared wavelength range.

EuS, on the other hand, enables simple fabrication of complex layer-based magneto-optic geometries. This is a very powerful and rare property among transparent magneto-optic materials. Since EuS is much cheaper than EuSe and also provides stronger MO response already at low magnetic fields, EuS could become a trigger for other interesting and potentially very powerful magneto-optic and magnetoplasmonic designs. This could include both 2D and 3D systems, for instance EuS photonic crystals, 2D nanopatches, as well as even magneto-optic metamaterials [20, 233].

The theory based on harmonic oscillators allows the description of the optical response of hybrid magnetoplasmonic systems in a simple and intuitive picture. This model provides analytical expressions for the optical response and confirms previous numerical and experimental findings. One important outcome of the model is that the spectral line shape of the MO response is fully determined by the optical properties of the system for zero magnetic field. Only the overall magnitude of the MO response is determined by the applied magnetic field. An extension of the oscillator model to 2D and 3D systems should be feasible. This could facilitate the development and optimization of magnetoplasmonic crystals of higher dimensions. Furthermore, by removing all plasmonic oscillators, the case of a purely dielectric grating-waveguide combination can be modeled. In prospect of non-linear magnetoplasmonics, the presented model could also be of fundamental relevance. Although the model is fully linear, a non-linear extension would be straightforward by adding higher order coupling terms [210].

Furthermore, as for most applications low insertion losses are crucial, the concept of EIT [215] could be a powerful mechanism to realize highly transparent magneto-optical nanophotonic systems. It would be especially interesting to see an experimental realization of magneto-optic EIT metasurfaces as theoretically demonstrated by Christofi et al [122] based on idealized material parameters.

Parts of this concluding section have been adapted with permission from the dissertation [43].

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References

[140] Chen J et al 2011 Plasmonic nickel nanoantennas Small 7 2341–7
[142] Bonanni V, Bonetti S, Pakizeh T, Pirzadeh Z, Chen J, Bonetti S, Pakizeh T, Pirzadeh Z, Chen J
[147] Li Y, Zhang Q, Nurmikko A V and Sun S 2005 Enhanced magneto-optical response in dumbbell-like Ag-CoFe 2O 4 nanoparticle pairs Nano Lett. 5 1689–92
[168] Ferreiro-Vila E et al 2009 Intertwined magneto-optical and plasmonic effects in Ag/Co/Ag layered structures Phys. Rev. B 80 125132


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